1) Understanding Equity Options
2) Setting up Brokerage Systems

M. Aras Orhan, 12.10.2013

FE 500 – Intro to Financial Engineering
Today’s agenda

- Basics of Equity Options
- A Synopsis of SDE based Pricing Models
  - Constant Volatility Models
    - Bachelier
    - Black, Scholes, Merton
    - “Skewness” in Volatility
- Setting up Brokerage Systems in Turkey
What is a derivative product?

- A derivative is a financial contract whose value depends on the value of an underlying asset.
- Underlying assets can be: equities, interest rates, foreign exchange, precious metals, or agricultural products, etc.
- Options are a type of derivative products.
- Other derivative products: swaps, futures, forwards, warrants, etc.
Typology of options

- Options
  - according to exercise term
    - American Types
    - European Types
  - according to payoff structure (flavors)
    - Vanilla Types
    - Exotic types
  - according to the direction of use
    - Call options
    - Put options
Equity options

- Equity option is an option contract that provides the buyer a right to buy/sell the underlying asset at a certain price, at or during a specific maturity.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Underlying Stock Name</th>
<th>Strike Price</th>
<th>Contract amount</th>
</tr>
</thead>
</table>

are named in the contract.

- If the option buyer decide "to use" the option, shares exchange with respect to the conditions of the contract. The decision to use, or not to use is completely under the responsibility of the buyer; the seller does not have any right on this issue.

- Seller takes the liability premium (option price) for this responsibility, and buyer pays this premium for the option he gets. This is indeed like an insurance contract (think of strike as ‘deductable’).
Quick facts about options

- You don’t have to own options to sell them. You can sell (write) an option contract and it is created and sold to a new buyer.

- Option payoffs are zero-sum. Ignoring the commissions paid to the market maker, the total amount created by issuing an option is zero. The buyer’s profit (loss) is the seller’s loss (profit).

- Options are cheaper than stocks. A short-maturity call option written for an ATM strike will likely sell for less than 10% of the underlying stock’s price.

- Whether it is American or European, the buyer can always exit her position by selling the option back anytime before maturity. Similarly a seller can always close her open position by buying one with same characteristics (strike and maturity).

- Both physical settlement (seller delivers or purchases the underlying at the strike price) or cash settlement (seller pays the option value at maturity to buyer) can occur, as defined in the contract.
Life of an ordinary option

Call Option:
- \(-C_0\)  \(-C_1\)
- Can be traded for price \(C_t\) at any time ‘t’ before maturity
- Buyer receives \((S_t - K)^+\)

Put Option:
- \(-P_0\)  \(-P_1\)
- Can be traded for price \(P_t\) at any time ‘t’ before maturity
- Buyer receives \((K - S_t)^+\)
The concept of “Moneyness”

- There is a special jargon to understand where the strike is located compared to the ongoing stock price.

- If at any point ‘t’ during the life of a call option,
  
  \[ S_t = K: \text{Option is said to be At The Money (ATM)} \]
  \[ S_t > K: \text{Option is In The Money (ITM)} \]
  \[ S_t < K: \text{Option is Out of The Money (OTM)} \]

  (if exercised now, it makes money)
  (if exercised now, it is worthless)

- Traders use this jargon fairly often.
Payoff graphs

Owner (Buyer) of European Call

Seller of European Call

Owner of European Put

Seller of European Put
Basic strategies - Covered Call

Definition:

Stock - Put = Covered Call
Basic strategies – Protective Put

**Definition:**

Stock + Put = Protective Put
Basic strategies – Straddle

Definition:
Call + Put = Straddle

Diagram:
- Long Put
- Long Call
- (Long) Straddle
- K
- S

P & L
Put – Call – Future relationship

Definition:

Call – Put = Future
Put – Call Parity

- $C - P = \text{Future}$  or
- $C - P = \text{Stock ( + borrowing)}$
Motives for trading options

- Hedging:
  e.g. Protective put

- Speculation:
  e.g. Straddle, Uncovered (Naked) Call - Put

- Market Making / Book running:
  Market maker provides liquidity and earns commissions and bid-ask spread.
Intrinsic and time values

Call Option Value

Value at issuance

Value at expiration

Price of underlying stock

K
Intrinsic and time values (cont’d)
Five variables

There are 5 variables that decide the value of a European Option, that is written on a non-dividend paying stock.

\[ C(S, K, t, r, \sigma) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Representation</th>
<th>How it affects value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Price</td>
<td>( S )</td>
<td>Intrinsic Value</td>
</tr>
<tr>
<td>Strike Price</td>
<td>( K )</td>
<td>Intrinsic Value</td>
</tr>
<tr>
<td>Time to Maturity</td>
<td>( t )</td>
<td>Time Value</td>
</tr>
<tr>
<td>Risk-free Rate</td>
<td>( r )</td>
<td>Time Value</td>
</tr>
<tr>
<td>Future Volatility</td>
<td>( \sigma )</td>
<td>Time Value</td>
</tr>
</tbody>
</table>
A synopsis of “closed-form” pricing in mathematical finance

- **Brownian Motion for Stock Price Dynamics** (Bachelier 1900, Samuelson 1965)
- **Square-root Process for Interest Rate Dynamics** (Cox, Ingersoll, Ross 1985)
- **Option Pricing Formula for European Vanilla Stock Options** (Black, Scholes 1973; Merton 1973)
- **Stochastic Volatility Model** (Heston, 1993)
- **Local Volatility** (Dupire 1994; Derman, Kani 1994)
- **Consistent Pricing of Exotic Options with more Coherent Volatility Surfaces of Vanilla Options**
Brownian Motion: Facts

“As the position of a particle spreads out with time, it is assumed that the variance of the increment is proportional to the length of time the Brownian motion has been observed.” (Wiersema, 2008)

- The increments are continuous
- Disjoint increments are independent of each other
- As time progresses the standard deviation of Brownian Motion increases, the density spreads out, and that probability is no longer negligible
ABM based models

- Bachelier (1900)
  Arithmetic Brownian Motion to model stock dynamics, and solved for option price
  \[ dS(t) = \mu dt + \sigma dB(t) \]

- Significance:
  - First SDE type model for asset prices

- Shortcomings:
  - Model allows negative stock prices
  - Model allows for negative option prices
  - No drift term, likely because it could not be ruled out from the pricing equation
GBM based models

- Kendall (1953), Roberts (1959), Osborne (1959; 1964) and Samuelson (1965)
  Geometric Brownian Motion to model stock dynamics

\[ dS(t) = \mu S(t)dt + \sigma S(t)dB(t) \]

- Sprenkle (1961; 1964), Boness (1964), Samuelson (1965), Samuelson and Merton (1969) solved for option prices using GBM
GBM based models (cont’d)

- **Significance:**
  - Model recognized risk aversion
  - Negative stock prices ruled out
  - Negative option prices are ruled out

- **Shortcoming:**
  - Could not rid of risk-aversion parameter or the drift term in pricing equation
Black, Scholes & Merton

Black and Scholes (1973), and Merton (1973), for the first time provided a pricing equation that was fully and instantaneously hedged, ruling out the drift term in the solution:

\[ \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + r S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} = r C \]

with boundary condition

\[ C(T, S) = (S - K)^{+} \]
Black, Scholes & Merton (cont’d)

- **Significance**
  - For the first time ever, the diffusion describing stock price movement was solved without the drift term in the solution.

- **Shortcomings**
  - Volatility is deterministic.
  - A single volatility for all option contracts based on the same underlying asset.
A look at the underlying dynamics

Equities

- ABM
  \[ dS(t) = \mu dt + \sigma dB(t) \]

- GBM
  \[ dS(t) = \mu S(t)dt + \sigma S(t)dB(t) \]

Interest Rates

- Ornstein-Uhlenbeck
  \[ dX(t) = -\lambda X(t)dt + \sigma dB(t) \]

- Mean Reversion SDE
  \[ dr(t) = -\lambda [r(t) - \bar{r}] dt + \sigma dB(t) \]

- Cox, Ingersoll, Ross
  \[ dr(t) = -\lambda [r(t) - \bar{r}] dt + \sigma \sqrt{r(t)} dB(t) \]
A more general model, with non-deterministic volatility

There are two problems inherent in the Black, Scholes, Merton model

- Instantaneous volatility is constant (dimensionless) in the model.
- Empirical stock returns are known to be heavy-tailed compared to the normal distribution.

More general models followed suit:

\[ dS(t) = \mu[t,S(t)]\,dt + \sigma[t,S(t)]\,dB(t) \]
Skewness

Implied vs. Lognormal
Volatility Surface

Practitioners, on the other hand, developed the concept of **Implied Volatility**, and started inferring the volatilities from the Black, Scholes, Merton model.

These market-inferred volatilities exhibited a skew in stock prices and a changing term structure.

The set of implied volatilities that yielded the market prices of options of different strikes and maturities came to be known as the **Volatility Surface**.
Implied Volatility Surface - Bloomberg

Source: Bloomberg LP
"TAKE-AWAYS" FROM TODAY’S LECTURE:

After the stock market crash in 1987, options prices at different strikes became increasingly mispriced by constant «Black-Scholes» volatility, as the market became increasingly sensitive to «tail risk».

Early pricing models for Vanilla Options have not modeled volatility in a conditional set-up. Need arose for models that were more consistent with the market prices.

Stochastic Volatility models were shown to explain, in a consistent way, why options with different strikes and expirations have different Black-Scholes implied volatilities.
Options pricing module - Bloomberg

Source: Bloomberg LP
Options pricing module - Bloomberg (cont’d)

Source: Bloomberg LP
Option trading in Turkish Equities

- OTC market (2006 – present)
- Warrants (2010 – present)
- Certificates (2012 – present)
- Exchange listed options: VIOP (21 Dec 2012 - present)
Suggested readings

- **Beginner**
  - Fundamentals of Futures and Options Market, John C. Hull

- **Intermediate**
  - Basic Black-Scholes: Option Pricing and Trading, Timothy Falcon Crack

- **Advanced**
  - Stochastic Calculus for Finance II: Continuous-Time Models, Steven E. Shreve
Setting up Brokerage Systems in Turkey

- **Equities**
  - Domestic OMS
  - International OMS

- **Listed Turkish Derivatives (VİOP)**

- **Leveraged FX**

- **OTC Equity Derivatives**

- **Listed Global Products**
Mehmet Aras Orhan
Director; Equity Derivatives, FX and Global Products

Work
Sept 2013 - Present, A Invest – Director; Equity Derivatives, FX and Global Products
  ▪ In charge of founding and managing the Derivatives, FX and Global Products brokerage businesses

2010 – Aug 2013, Yapı Kredi Invest - Head of Product Development and Projects
  ▪ FIX DMA, Algorithmic Trading, OTC Turkish Derivatives (Murex), OTC Leveraged FX, Issuance of Warrants

Fields of Interest & Study
  ▪ Empirical Asset Pricing (emphasis on Volatility Modeling and Estimation)
  ▪ High Frequency Market Dynamics
  ▪ Equity Derivatives (designed and coded a pricing and middle-office/risk assessment component in VBA, currently at use at Yapı Kredi Bank)
  ▪ Program Trading (experience on evaluation of algorithmic trading platforms)
  ▪ Trading Systems (multi-asset platforms, with an in-depth knowledge of the FX system setup)

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