

Fuzzy Boundary Layer Tuning as Applied to the Control of a Direct Drive Robot

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Abstract - Chattering in the control signal is a significant problem in sliding mode control applications. The boundary layer approach is one of the many modifications proposed in the literature to avoid the chattering. In this approach, instead of the discontinuous sliding mode control, a high gain feedback control law is employed. The thickness of the boundary layer is an important design parameter. This paper proposes a fuzzy on-line tuning method to adjust the boundary layer thickness for the best system performance without chattering. The method features the measurement of the chattering in the control signal. Experimental results with a two degrees of freedom direct drive robot arm are presented.

I. INTRODUCTION

In the robotic manipulator control, where coupled, non-linear system dynamics and varying system parameters arise as difficulties, Sliding Mode Control (SMC) [1-4] has been proposed as a robust control strategy. The primary characteristic of SMC is that the control signal is discontinuous. When certain conditions are met, a sliding motion on a predefined subspace of the state-space is established in which the system state repeatedly crosses the switching surface. A number of works are reported on the discrete time implementation and stability analysis of sliding mode controllers [5-9]. [9] suggests that the continuous time stability considerations can still be valid for discrete time systems, provided that the sampling period is chosen short enough.

The high speed (ideally at infinite frequency) switching necessary for the establishment of a sliding mode causes oscillations about the sliding surface. In sliding mode motion control applications, high frequency oscillations of the controller output, termed as chattering, introduce a problem. A thorough explanation of the problem and what may cause it is given in [2]. In practical implementations, chattering is highly undesirable because it may excite unmodeled high frequency plant dynamics and this can result in unforeseen instabilities.

To overcome this problem, many modifications to the original sliding control law have been suggested [10], [11]. The most popular modification is the boundary layer approach, which is, basically, the application of a high gain feedback when the motion of the system reaches β -vicinity of a sliding manifold [4]. This approach is based on the idea of the equivalence of the high gain systems and the systems with sliding modes [12]. In this method, the determination of

an appropriate value for the boundary layer thickness is significant. Too narrow boundary layers cannot solve the chattering problem, whereas, too large values for the boundary layer cause degradation in tracking performance and robustness.

With the development of the intelligent control field, new control approaches based on fuzzy logic, neural networks and evolutionary computing have come into common use. These control methodologies provide an extensive freedom for control engineers to exploit their understanding of the problem, to deal with problems of vagueness, uncertainty or imprecision and learn from experience. Fuzzy control systems, as tools against the problems of uncertainty and vagueness, incorporate human experience into the task of controlling a plant. When employed in robotic trajectory control, they mainly play one of two roles in the controller structure. One of them is to compute the control signal by fuzzy rules. The other one is to tune, adapt or schedule the parameters of other control mechanisms to accomplish better performance in face of uncertainties and different operating points. The combination of sliding mode controllers and fuzzy logic systems are studied by a number of researchers and a recent survey on this field is presented in [13].

This paper proposes an on-line fuzzy tuning algorithm for the boundary layer thickness. The method is based on the introduction of a "chattering variable" [14-16]. The fuzzy system uses the chattering variable and the sliding variable as its inputs to determine a feasible thickness for the boundary layer. The tuning scheme is similar to the one introduced in [16]. However, the present paper employs different membership functions and it also validates the method through experiments.

The next section outlines the design of a classical sliding mode controller with a boundary layer for second order SISO systems as a basis of the discussion in later sections. The third section presents the on-line tuning method. The experimental results obtained with a two-dof direct drive arm, are presented in Section IV. The conclusion is presented in the fifth section.

II. CLASSICAL SMC WITH A BOUNDARY LAYER

In this section, classical sliding mode controller design is reviewed. For simplicity, second order SISO systems are focused on. The controller is derived for systems with the following form of state space equations.

$$\ddot{x} = f(X) + b(X)u \quad (1)$$

Here, X is an augmented vector of the state variables defined as

$$X = [x \quad \dot{x}]^T \quad (2)$$

u is the control input and the input gain b takes strictly positive values. The tracking error is defined by

$$e = x_d - x \quad (3)$$

where x_d is the desired value for x . In sliding mode control of second order systems, commonly, the desired dynamic response for this system is given by

$$s(e) = \dot{e} + \lambda e = 0 \quad (4)$$

where λ is a positive number for stability. If s can be forced to zero, the desired dynamics can be attained and the tracking error will converge to zero with the dynamics dictated by (4) which corresponds to a line with slope $-\lambda$ in the phase plane as shown in Fig. 1. An approach which involves the selection of a Lyapunov function V of s is followed in the literature mostly and this function is chosen as

$$V = \frac{1}{2} s^2 \quad (5)$$

The control law is constructed such that the sliding line is attractive for the state trajectories on the phase plane.

This can be achieved if \dot{V} is negative definite i.e. if it assumes negative values except in the origin. A choice for a negative definite \dot{V} is given in the following equation

$$\dot{V} = s\dot{s} = -K'|s| \quad (6)$$

where K' is a positive constant. Defining ϕ as

$$\phi(t) = \dot{x}_d + \lambda x_d \quad (7)$$

from (1-4), it can be noted that this equation is equivalent to

$$s(\dot{\phi}(t) - f(X) - b(X)u - \lambda\dot{x}) = -K'|s| \quad (8)$$

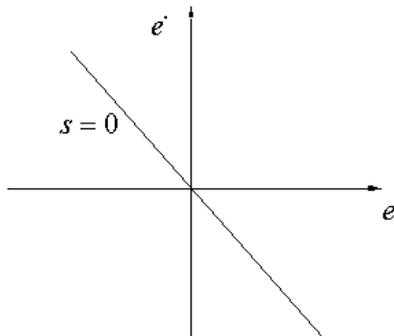


Fig.1. The sliding line

Hence, negative definiteness of \dot{V} can be achieved by selecting u as

$$u = \frac{1}{b}(\dot{\phi} - f - \lambda\dot{x}) + \frac{K'}{b} \text{sign}(s) \quad (9)$$

Here, $\text{sign}(s)$ is the signum function defined by

$$\text{sign}(s) = \begin{cases} -1 & \text{if } s < 0 \\ 0 & \text{if } s = 0 \\ 1 & \text{if } s > 0 \end{cases} \quad (10)$$

In practice, the functions f and b cannot be known exactly and therefore their estimates are used instead. The control law becomes then

$$u = \frac{1}{\hat{b}}(\dot{\phi} - \hat{f} - \lambda\dot{x}) + K \text{sign}(s) \quad (11)$$

where $K = K'/\hat{b}$. If the bound of the uncertainties on f and b are known the gain K can be selected adequately high to assure robustness in face of these uncertainties. The signum function requires infinite switching frequency, in the theory, to keep the system states on the sliding line. However, because of some factors like actuator limitations and delays which are inevitable when the controller is implemented on digital computers, infinite frequency switching cannot be realized. As a result, frequent state trajectory jumps across the sliding line are observed. Such frequent jumps are called chattering in the sliding mode control terminology. It can also be identified high frequency and high amplitude oscillations in the control input and velocity signals in the motion control field. Many modifications of the original control law are proposed to alleviate the chattering problem. The most simple and popular solution is the so called boundary layer approach in which the signum function is replaced by the saturation function (Fig. 2)

$$\text{sat}(s) = \begin{cases} -1 & \text{if } s \leq -\beta \\ s/\beta & \text{if } -\beta < s \leq \beta \\ 1 & \text{if } \beta < s \end{cases} \quad (12)$$

to obtain the control law

$$u = \frac{1}{\hat{b}}(\dot{\phi} - \hat{f} - \lambda\dot{x}) + K \text{sat}(s) \quad (13)$$

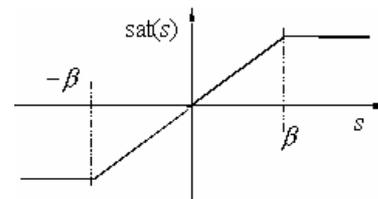


Fig. 2. The saturation function

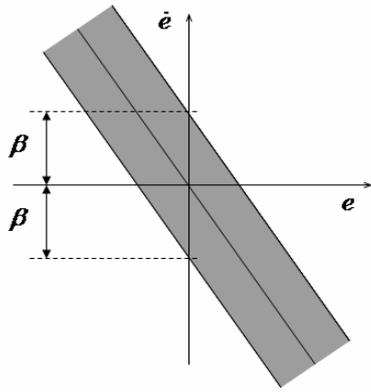


Fig. 3. The boundary layer

The parameter β defines the thickness of the boundary layer as shown in Fig. 3.

III. THE ON-LINE TUNING METHOD

In practical applications where the controller is implemented on digital computers, infinite frequency switching is not possible, simply because of the time delays introduced by the sampling time. Therefore a zigzag behavior is observed around the sliding line, rather than an exact following of it. A number of papers which investigate the sliding mode controllers in discrete time applications appeared in the literature and discrete time equivalents of the stability analysis tools in continuous time are sought [5-9]. Generally, the conditions which will assure stability in discrete time are in the form of inequalities involving the sampled sliding variable and the control laws governed by such considerations are also discrete time equations unlike the ones in the previous section. It is also proposed that control laws obtained in continuous time can still assure stability provided that the sampling period is selected shorter than a certain limit [9]. The controller designed in the previous section with considerations for continuous time is studied in this paper. However, implementation on digital computers is assumed and its effects are investigated.

In the digital implementation, even when the classical sliding mode controller with the signum function is employed, tracking performance cannot be guaranteed. The zig-zag behavior is observed even for small values of the boundary layer thickness β . For larger values of β chattering disappears, however, with increasing β , the tracking performance deteriorates. This suggests that there is a critical value of β , such that below this value chattering occurs and above this value the tracking performance becomes worse. Further, the work in [15] suggests that the minimum attainable peak tracking error for a set of sinusoidal reference trajectories is obtained at this value of β (Fig. 4). In the boundary layer approach, this puts the limits of the achievable performance, assuming that all other controller parameters, including the sliding line slope, the gain parameter K and the computation of the equivalent control, are kept the same.

Such a limiting value for the thickness of the boundary level can be found experimentally by trial and error. However, this value may be highly dependent on the

reference trajectory used in the experimental tuning. Further, in the case of real implementation, during the tuning process, the mechanical system can observe high amplitude chattering which is undesired. It can also be stated that slight changes in the plant dynamics can push the equilibrium to the chattering side.

Therefore, the value of the boundary layer thickness should be varying according to the chattering level in the control signal in order to achieve the best performance possible.

To this end, in equation (13), a measure of chattering, Γ , is introduced.

$$\Gamma = |\dot{u}| \quad (14)$$

$|\dot{u}|$, the absolute value of the derivative of the control signal is obtained by Euler approximation in the digital implementation. Fig. 5 illustrates typical behaviour of this detector in the case of too much control activity or ringing in the control signal.

A variety of chattering measures can be formulated. Similar measures of chattering are used in [14] and [15] for the on-line tuning of control parameters of sliding mode controllers. Equipped with such a measure of chattering, many parameter adjustment methods which relate the boundary layer thickness β to the control activity can be devised. The main idea can be summarized as below.

(i) When chattering occurs, the boundary thickness should be increased to force the control input to be smoother.

(ii) The boundary layer thickness should be decreased if the control activity is low. This is since, in order to obtain best tracking performance some amount of activity in control is needed and our aim is to operate the system at the limit of chattering. Low control activity can be identified by small values of the chattering variable Γ .

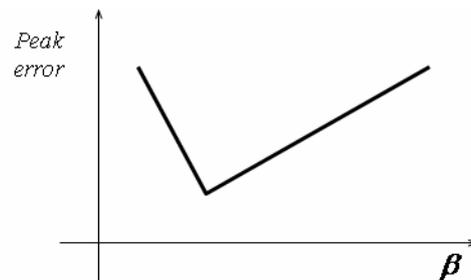


Fig. 4. Typical boundary layer thickness versus error relation

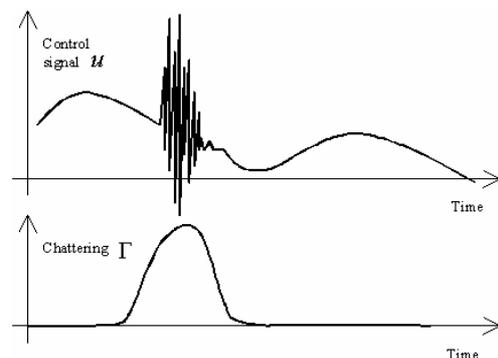


Fig. 5. The chattering variable

The guidelines (i) and (ii) on their own can be used to devise boundary layer adjustment methods [15]. However, they use only the information about the chattering in the system. Another source of valuable information is the sliding variable. The following guidelines describe the role of the sliding variable in the boundary layer adjustment used in this paper.

(iii) When the absolute value of the sliding variable is low, the phase trajectory is close to the sliding line and hence a steep saturation function (narrow boundary layer) is likely to introduce chattering effect.

(iv) When the absolute value of the sliding variable is high, the phase trajectory is far away from the sliding line and hence a steep saturation function is desirable to decrease the duration of the reaching phase.

The four guidelines above can be used in various ways to construct a tuning mechanism for β . An analytic relation between Γ , s and β can be one of the choices. The method proposed in this paper employs fuzzy systems for the on-line tuning of β . Fuzzy systems, as also mentioned previously, are natural choices to exploit verbal descriptions (like the four guidelines above) of the plant or problem to obtain control or adaptation mechanisms.

Tables I and Fig. 6 describe the four fuzzy rules used in the tuning. In Table I “NB” stands for negative big, “NS” is negative small and PB is positive big. The defuzzification is carried out by the following expression.

$$\Delta\beta = \frac{\mu_{Big\ s}\mu_{Small\ \Gamma}\Delta\beta_{NB} + \mu_{Big\ s}\mu_{Big\ \Gamma}\Delta\beta_{NS} + \mu_{Small\ s}\mu_{Big\ \Gamma}\Delta\beta_{BP}}{\mu_{Big\ s}\mu_{Small\ \Gamma} + \mu_{Big\ s}\mu_{Big\ \Gamma} + \mu_{Small\ s}\mu_{Small\ \Gamma} + \mu_{Small\ s}\mu_{Big\ \Gamma}} \quad (15)$$

This corresponds to a singleton defuzzification mechanism. Finally, β is obtained by

$$\beta(k) = \beta(k-1) + \Delta\beta(k-1) \quad (16)$$

at every control cycle k .

TABLE I
THE FUZZY RULE BASE

		Γ	
		Small Γ	Big Γ
$ s $	Big $ s $	NB $\Delta\beta$	NS $\Delta\beta$
	Small $ s $	0	PB $\Delta\beta$

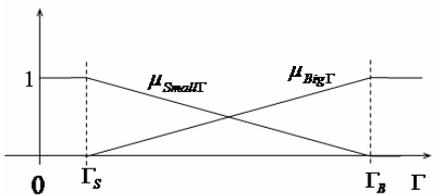
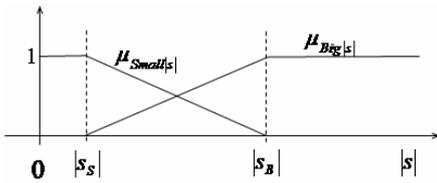


Fig. 6. The membership functions

It can easily be noted that the choice of the rule base satisfies the conditions (i) - (iv) above. In the next section, simulation studies with this on-line tuning rule are presented.

IV. EXPERIMENTAL RESULTS

A two-dof direct drive manipulator built at Sabanci University Robotics Laboratory is shown in Fig. 7. This manipulator is used as the test bed in the experimental studies. Link mass and length parameters of this manipulator are given in Table II. The Yokogawa Dynaserv direct drive motors used at base and elbow joints provide position signals with a resolution of 1024000 pulses/rev. In the following, the chattering-free sliding-mode algorithm is applied to the control of the direct-drive manipulator. The base motor torque capacity is 200 Nm and that of the elbow motor is 40 Nm. A dSpace 1102 DSP-based system is used to control the arm. The user interface is on a Pentium 3 PC machine. C language servo routines are compiled in this environment and downloaded to the DSP.

The dynamics equation of the robot can be expressed as

$$\left(\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} + D(q_1, q_2) \right) \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \left(C(q_1, q_2, \dot{q}_1, \dot{q}_2) + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \right) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} F_{c1} \\ F_{c2} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (17)$$

q_1 is the base joint angular position and q_2 the angular position of the elbow.

J_1 and J_2 are the rotor inertia values for the base and elbow joints, respectively. D is the manipulator inertia matrix. C represents the matrix for centripetal and Coriolis effects, B_1 and B_2 are constant viscous friction coefficients for the two joints. F_{c1} and F_{c2} stand for the Coulomb friction torques. The robot is controlled by the joint actuation torques u_1 and u_2 . Because of the horizontal kinematic arrangement of the links there is no gravity effect acting on the joints.



Fig. 7. The direct drive SCARA type robot arm

TABLE II
LINK MASSES AND LENGTHS OF THE ARM

Link 1(upper arm) weight	23 kg
Link 2 (elbow link) weight	4 kg
Link 1(upper arm) lenght	0.4 m
Link 2 (elbow link) length	0.32 m

The matrices D and C are obtained with the Euler-Lagrange method [17] by using the various mass, length, and inertia parameters of the arm and the direct-drive motors. The values for J_1 and J_2 are tabulated in the manufacturers documentation. Friction parameters, especially Coulomb friction, however, are difficult to model.

The sliding controller for this system is obtained by applying (13) to the robot joints independently. Decoupled links are assumed and simplified SISO equations of motion are obtained with fixed effective inertia and damping parameters, as follows.

$$\begin{aligned} (J_1 + D_{11\text{-nominal}})\ddot{q}_1 + \hat{B}_1\dot{q}_1 &= u_1 \\ (J_2 + D_{22\text{-nominal}})\ddot{q}_2 + \hat{B}_2\dot{q}_2 &= u_2 \end{aligned} \quad (18)$$

where

$$J_{eff_1} = (J_1 + D_{11\text{-nominal}}), \quad J_{eff_2} = (J_2 + D_{22\text{-nominal}}). \quad (19)$$

In the equations above, $D_{11\text{-nominal}}$ and $D_{22\text{-nominal}}$ are diagonal entries of the inertia matrix $D(q_1, q_2)$ computed at a configuration corresponding to a stretched elbow. \hat{B}_1 and \hat{B}_2 are estimates of viscous friction coefficients obtained through experiments. Centripetal and Coriolis effects, coupling between the joints, and Coulomb friction are omitted in the equations. Only very limited knowledge on the robot dynamics is used.

It can be noted that the equations in (18) can be expressed in the form (1) too:

$$\begin{aligned} \ddot{q}_1 &= -\frac{\hat{B}_1}{J_{eff_1}}\dot{q}_1 + \frac{1}{J_{eff_1}}u_1 \\ \ddot{q}_2 &= -\frac{\hat{B}_2}{J_{eff_2}}\dot{q}_2 + \frac{1}{J_{eff_2}}u_2 \end{aligned} \quad (19)$$

The control law (13) is then applied as

$$\begin{aligned} u_1 &= \frac{1}{\hat{b}_1}(\dot{\phi}_1 - \hat{f}_1 - \lambda_1\dot{x}_1) + K_1\text{sat}(s_1), \\ u_2 &= \frac{1}{\hat{b}_2}(\dot{\phi}_2 - \hat{f}_2 - \lambda_2\dot{x}_2) + K_2\text{sat}(s_2) \end{aligned} \quad (20)$$

with

$$\begin{aligned} x_1 &= q_1, \quad \hat{f}_1 = -\frac{\hat{B}_1}{J_{eff_1}}\dot{q}_1, \quad \hat{b}_1 = \frac{1}{J_{eff_1}}, \\ x_2 &= q_2, \quad \hat{f}_2 = -\frac{\hat{B}_2}{J_{eff_2}}\dot{q}_2, \quad \hat{b}_2 = \frac{1}{J_{eff_2}}. \end{aligned} \quad (20)$$

The controller parameters and the parameter values for the fuzzy tuning law in (15) are shown in Table III. A control cycle time of 1 ms is used. The position reference trajectory represents step joint references of 0.1 rad applied to the two joints. The initial condition corresponds to a stationary pose with extended elbow.

The control signal, the joint position error and the β parameter curve of the elbow joint are presented in Fig.8. A steady state error of 3.0×10^{-5} rad is obtained. Similar results are obtained for the base joint. However, they are not presented in this paper due to lack of space.

It can be observed that too high control activity is responded by sudden increase in the boundary layer thickness. As a result the control signal becomes smoother, and chattering decreases.

Note that, with the membership functions in Fig. 6 and the rules in Table I, the no change in the β -parameter is done if the absolute value of the sliding variable is less than $|s_s|$ and the chattering variable is less than Γ_s . Therefore, the β -parameter quickly converges when desired distance to the sliding line is reached with an acceptable chattering level.

The phase plane trajectory and the sliding line for the shoulder are shown in Fig. 9. The sliding line overshoot, which is commonly observed with SMC systems “oversmoothed” by too wide boundary layers is not the case. The sliding line is not followed exactly. However, it is tracked closely and the desired error dynamics is achieved to a good approximation. This indicates that the on-line tuning algorithm for the boundary layer thickness is successful. The results obtained in [16] show less chattering than the ones presented in this paper. However, we believe that this is because [16] presents simulation results and the simulation model in [16] does not include Coulomb friction effects.

TABLE III
CONTROLLER PARAMETERS

Parameter	Value	Parameter	Value
K_1	30	$ s_B $	4
K_2	15	$ s_S $	0.01
λ_1	50	$\Delta\beta_{BP}$	0.1
λ_2	80	$\Delta\beta_{NB}$	-0.1
Γ_B	200000	$\Delta\beta_{NS}$	-0.05
Γ_S	2000		

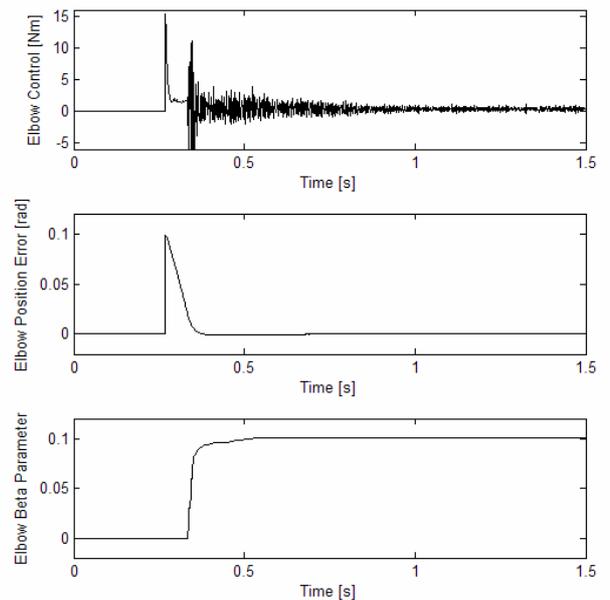


Fig. 8. Control and error signals and the boundary layer thickness parameter for the elbow joint. The steady state error is 3×10^{-5} rad.

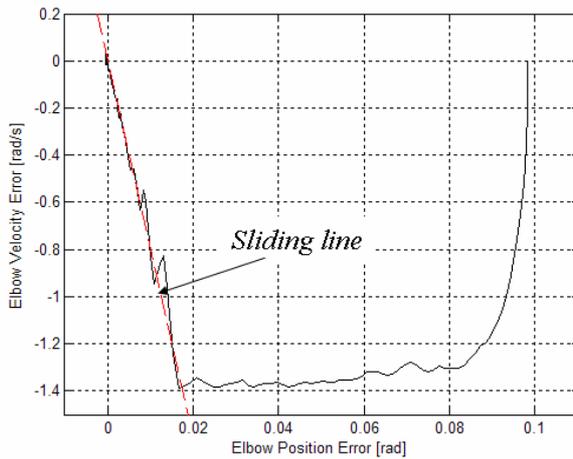


Fig. 9. Phase plane trajectory for the elbow joint

V. CONCLUSION

Sliding mode controller structures employing boundary layers are intensively studied in the literature. This approach solves the chattering problem, however tracking performance is degraded when compared with the original sliding mode algorithm. The determination of a suitable boundary layer thickness which can achieve best performance and still eliminate chattering is an important design issue. This paper proposes an on-line fuzzy tuning method to adjust the thickness of the boundary layer.

The thickness of the boundary layer is obtained without the off-line trial and error processes. The on-line algorithm reacts to changes in the sliding variable and the chattering level to adjust the boundary layer thickness continuously. It is demonstrated by experiments that the performance and chattering elimination properties of the proposed method make it a candidate for industrial applications.

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