Problem
Consider a packet switched network with two intermediate nodes between sender A and receiver B (that is, A → x → y → B). Define C as the capacity of the links in bps. Define D as the bit size of application data. Define H as the header size in bits. Define P as the number of packets for this application data. Assume no processing and propagation delays. Nodes receive the whole packet and then transmit to the next node.

(a) Derive a formula for the end-to-end delay for that particular application data.
(b) Given that C = 5,000 bps, D = 50,000 bits, H = 500 bits, what would be the optimum number of packets that minimizes the total end-to-end delay? Calculate the delay too.

Solution

(a) \[ T_{\text{end-to-end}} = \frac{\text{Total #of bits transmitted}}{\text{Channel Capacity}} \]

Since D is the total size of application data, and P is the packet number, each packet consists of \(\frac{D}{P}\) bits. The first packet arrives with a three packets delay. The total number of packets on the wire will be \((P+2)\)

Time necessary to send one packet is:
\[ \frac{(H + \left(\frac{D}{P}\right))}{C} \]

Then total time necessary to send \((P+2)\) packets will be:
\[ T_{\text{end-to-end}} = \frac{(P+2)\left(\frac{(H + \left(\frac{D}{P}\right))}{C}\right)}{C} \]

(b) Given that:
\[ C = 5000 \text{ bps}, \quad D = 50000 \text{ bits}, \quad H = 500 \text{ bits}. \]

What is the optimum number of packets \(P_{\text{opt}}\)?

Substituting the given values, the end-to-end delay formula is derived as below:
\[ T_{\text{end-to-end}} = \frac{(P+2)\left(\frac{(500 + (50000/P))}{5000}\right)}{5000} \]
\[ T_{\text{end-to-end}} = \frac{(P+2)\left(1/10 + (10/P)\right)}{5000} \]
\[ T_{\text{end-to-end}} = \frac{(P+2)/10 + 20/P + 10}{5000} \]

The optimum number of packets \(P_{\text{opt}}\) can be find by taking the derivative of the formula below and find the point \(P\) where the formula takes the minimum value.

\[ \frac{dT_{\text{end-to-end}}}{dp} = 1/10 - 20/P^2 \rightarrow P^2 = 200 \rightarrow P \approx 14.14 \]

We need an integer value for the number of packets. In this case we have two options, \(P = 14\) and \(P = 15\). Let’s calculate the end-to-end delay times for these values. The minimum one will be the optimum number of packets.

For \(P = 14\),
\[ T_{\text{end-to-end}} = \frac{(P+2)/10 + (20/P) + 10}{5000} = 1.6 + 1.428 + 10 = 13.028 \]

For \(P = 15\),
\[ T_{\text{end-to-end}} = \frac{(P+2)/10 + (20/P) + 10}{5000} = 1.7 + 1.176 + 10 = 13.033 \]

Therefore, we can choose \(P_{\text{opt}} = 14\)