Linear Programming and Extensions

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Office Hours: M 11:30-12:00, T 10:30-12:00
Class Hours: M 10:40-11:30 (FENS G032), T 08:40-10:30 (FENS L063)

Course Description

This course focuses on the theory of linear programming (LP) although we will discuss some practical implementation issues as well. In the first part of the course, we will review LP modeling, convexity, some essential linear algebra, certain aspects of polyhedral theory, and then present the foundations of linear programming. In the second part of the course, we will discuss the simplex method for solving linear programs and its algorithmic aspects, some related computational issues, duality and sensitivity. Finally, we will cover Dantzig-Wolfe decomposition and column generation for solving large-scale linear programs. If time permits at the end of the semester, we will briefly introduce further decomposition methods that rely on linear programming and interior point algorithms for solving linear programs. Issues related to computer implementations of linear programming algorithms will generally not be discussed in class, but you will be exposed to the computational aspects of linear programming during the term project.

Course Outline

1. Introduction.
   (a) The linear programming problem.
   (b) Linear programming examples.
   (c) Geometric solution to the two-dimensional problem.

2. Linear algebra, convexity, polyhedral sets.
   (a) Review of useful linear algebra.
   (b) Convex sets and convex functions.
   (c) Polyhedral sets and polyhedral cones.

   (a) Separating hyperplane theorem for cones and its implications.
4. The simplex method.
   (a) Basic sequences and basic solutions.
   (b) Feasibility/infeasibility, boundedness/unboundedness, optimality/non-optimality certificates based on basic sequences.
   (c) Simplex method (under the assumption of a starting feasible basic sequence and non-degeneracy).
   (d) Proof of finite convergence for the simplex method under non-degeneracy.
   (e) Obtaining an initial basic feasible solution.
   (f) Degeneracy and finite convergence under degeneracy.

5. Duality theory and sensitivity analysis.
   (a) Weak and strong duality theorems.
   (b) Primal and dual relationships.
   (c) Dual simplex method.
   (d) Post-optimality and sensitivity analysis.
   (e) Parametric analysis.
   (f) Economic interpretation of the dual.

6. Decomposition methods.
   (a) Dantzig-Wolfe decomposition and column generation.
   (b) Lagrangian relaxation (if time permits).
   (c) Benders decomposition (if time permits).

7. Introduction to interior point algorithms (if time permits).

Grading

1. Homeworks will be assigned regularly. Solutions will be provided, but homeworks will not be graded. Note that you still have to turn in your homeworks.

2. Four pop-up quizzes (20% total). The worst grade will be dropped. The quizzes will be closely related to the homework questions.

3. Midterm (20%).

4. Final (40%).

5. Project (20%).
Software

Information and support for all academic software provided by the university can be found at this link.

1. Some homework problems and the project may require formulating an LP model and actually solving it with a computer program. You are free to use any software that you like (IBM ILOG CPLEX Optimization Studio, MATLAB, GAMS, AMPL, LINDO/LINGO, etc.), but the solutions provided will either employ IBM ILOG CPLEX Optimization Studio or MATLAB. It is your responsibility to learn how to use these software. The installation file of IBM ILOG CPLEX Optimization Studio can be downloaded from the software server of the university:

   \software\academic\ILOG

   These are full versions that do not require a license key under the Academic Initiative of IBM. I use version 12.6.1.

2. For matrix computations, MATLAB is very useful, and it does also have an interface to the CPLEX libraries. That is, you can solve linear programs directly from within MATLAB by invoking CPLEX. You will also need MATLAB for the term project. Therefore, it is highly recommended that you familiarize yourself with it. You can find MATLAB under:

   \software\academic\Matlab

   The detailed installation instructions are at this link, and further information can be obtained from MathWorks.

Textbook


References

Introduction to Linear Optimization, Dimitris Bertsimas, John N. Tsitsiklis. Athena Scientific, 1997. (This book is on reserve at the Information Center.)


Academic Integrity

Please be aware that violations of academic integrity will be subject to disciplinary action. I strongly advise you to go through the academic integrity policy implemented at Sabancı University. This policy as well as the related announcements can be accessed through the internal website of the university.