Abstract— Humanoid robotics attracted the attention of many researchers in the past 35 years. The motivation of research is the suitability of the biped structure for tasks in the human environment. The control of a biped humanoid is a challenging task due to the hard-to-stabilize dynamics.

Walking reference trajectory generation is a key problem. A criterion used for the reference generation is that the reference trajectory should be suitable to be followed by the robot with its natural dynamics with minimal control intervention. Reference generation techniques with the so-called Linear Inverted Pendulum Model (LIPM) are based on this idea. The Zero Moment Point (ZMP) Criterion is widely employed in the stability analysis of biped robot walk. Improved versions of the LIPM based reference generation obtained by applying the ZMP Criterion are reported too. In these methods, the ZMP during a stepping motion is kept fixed in the middle of the supporting foot sole. This kind of reference generation lacks naturalness, in that, the ZMP in the human walk does not stay fixed, but it moves forward, under the supporting foot.

This paper proposes a reference generation algorithm based on the LIPM and moving support foot ZMP references. The application of Fourier series approximation simplifies the solution and it generates a smooth ZMP reference. Trajectory and force control methods for locomotion are devised and applied too. The developed techniques are tested through simulation with a 12 DOF biped robot model. The results obtained are promising for implementations.

I. INTRODUCTION

Humanoid robotics attracted the attention of many researchers in the past 35 years. It is currently one of the most exciting topics in the field of robotics and there are many projects on this topic [1-6]. The motivation of research is the suitability of the biped structure for tasks in the human environment and the goal of the studies in this area is to reach the human walking dexterity, efficiency, stability, effectiveness and flexibility.

The control of a biped humanoid is a challenging task due to the many degrees of freedom involved and the non-linear and hard to stabilize dynamics [7-8].

Walking reference trajectory generation is a key problem. Methods ranging from trial and error to the use of optimization techniques with energy or control effort minimization constraints are applied as solutions.

A very intuitive criterion used for the reference generation is that the reference trajectory should be suitable to be followed by the robot with its natural dynamics, without the use of extensive control intervention. Reference generation techniques with the so-called Linear Inverted Pendulum Model are based on this idea [9-10]. Simply stated, the walking cycle is then achieved by letting the robot start falling into the walking direction and to switch supporting legs to avoid the complete falling of the robot.

Yet another intuitive demand for the biped robot reference generation is that the reference trajectory should be a stable one, in the sense that it should not lead to unrecoverable falling motion. The Zero Moment Point Criterion [7] introduced to the robotics literature in early 1970s is widely employed in the stability analysis of biped robot walk. Improved versions of the Linear Inverted Pendulum Model based reference generation, obtained by applying the Zero Moment Point criterion in the design process, are reported too [11]. In this approach the Zero Moment Point during a stepping motion is kept fixed in the middle of the supporting foot sole for the stability, while the robot center of mass is following the Linear Inverted Pendulum path.

Although reference generation with the Linear Inverted Pendulum Model and fixed Zero Moment Point reference positions is the technique employed for the most successful biped robots today, this kind of reference generation lacks naturalness at one point. Investigations revealed that the Zero Moment Point in the human walk does not stay fixed under the supporting foot. Rather, it moves forward from the heel to the toe direction [12-14].

In [14] Zhu et. al propose this idea of using variable ZMP to generate a dynamically stable gait in terms of linear inverted pendulum approach. They consider it to follow first order functions from the heel to toe of the foot in single support phase.

This paper takes a similar approach and proposes a reference generation technique based on the Linear Inverted Pendulum Model and moving support foot Zero Moment Point references. The application of Fourier series approximation to the solutions of the Linear Inverted Pendulum dynamics equations simplifies the solution as in [15], and it generates a smooth Zero Moment Point reference for the double support phase. Foot reference trajectory generation methods for smooth swing foot trajectories, trajectory control methods for the center of mass of the robot and force control techniques for the landing foot are employed in this paper too.

The reference generation and control techniques are simulated and animated in a 3-D full dynamics simulation.
environment with a 12 DOF biped robot model. The results obtained are promising for implementations.

The biped model used as the test bed for the developed techniques is introduced in Section II. The reference generation with natural moving ZMP trajectories and the control of locomotion are discussed in Sections III and IV, respectively. Section V presents the simulation results and their analysis. The conclusion is drawn lastly.

II. THE BIPED ROBOT MODEL

The biped model used in this paper consists of two 6-DOF legs and a trunk connecting them (Fig. 1). Three joint axes are positioned at the hip. Two joints are at the ankle and one at the knee. Link sizes and the masses of the biped are given in Table I.

<table>
<thead>
<tr>
<th>A. Link</th>
<th>Dimensions (LxWxH) [m]</th>
<th>Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>0.2 x 0.4 x 0.5</td>
<td>50</td>
</tr>
<tr>
<td>Thigh</td>
<td>0.27 x 0.1 x 0.1</td>
<td>12</td>
</tr>
<tr>
<td>Calf</td>
<td>0.22 x 0.05 x 0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Foot</td>
<td>0.25 x 0.12 x 0.1</td>
<td>5.5</td>
</tr>
</tbody>
</table>

TABLE I MASSES AND DIMENSIONS OF THE ROBOT LINKS

III. REFERENCE GENERATION WITH NATURAL ZMP TRAJECTORIES

LIPM mode approach is based on such ordinary differential equations that the solutions are both hard to be solved and they are composed of numerically unbounded \( \cosh() \) functions. In addition they are sensitive to the height variation of the pendulum and they are difficult to be used robustly. Furthermore, since only the acceleration of the body is considered in LIPM approach the foot stepping positions may vary as a result. However, the stepping positions in real implementations are generally determined by exogenous environmental needs. For instance a robot should determine its foot stepping positions in order to avoid obstacles in real experiments. As a result the robot should have such a gait that follows the pre-determined stepping positions and preserve the overall stability. As a solution to such problems Choi, Y. et. al [15] introduce an alternative robust CoM trajectory planning method by using the approximate solution composed of bounded functions. Having pre-determined ZMP reference trajectories Choi, Y. et. al find the exact solutions of LIPM equations that are derived according to ZMP criterion. Finally they derive the approximated closed form equations that give the time trajectory of the CoM.

However in their studies Choi, Y. et. al use fixed ZMP trajectories. This actually leads the robot walking both to be rigid and unnatural. Furthermore, in their approximated solutions they do not consider double support phases which, eventually, may bring problems in real implementations. In this section an approximation to the solution of the dynamics of LIPM by considering natural ZMP references with double support phase is introduced.

A. Linear Inverted Pendulum Model

The main idea of the LIPM approach [9] is to extract a dominant feature of biped dynamics, which is high-order and non-linear, and to use this dominant factor to explain the governing dynamics of the system. In this model the robots mass is assumed to be lumped at the center of mass of the robot and the legs of the robot are assumed to be massless. Further, for simplicity, the height of the pendulum is assumed to be constant in this model. This lets the dynamics of the model to be linear. Such an inverted pendulum with a massless rod can be seen in Fig. 2 where \( C = [c_x, c_y, c_z] \).

The ZMP relations for \( x - y \) plane are as follows.

\[
\begin{align*}
    x_{\text{zmp}} &= \sum_{i=1}^{n} m_i (\ddot{z}_i - g_z) \dot{x}_i - \sum_{i=1}^{n} m_i (\ddot{x}_i - g_x) \dot{z}_i \\
    y_{\text{zmp}} &= \sum_{i=1}^{n} m_i (\ddot{y}_i - g_y) \dot{x}_i - \sum_{i=1}^{n} m_i (\ddot{x}_i - g_x) \dot{y}_i 
\end{align*}
\] (1)
Let the ZMP of coordinates of this pendulum to be 
\[ P = [p_x, p_y, p_z]^T, \]
the mass of the pendulum to be \( m \).
The gravity vector is \( \hat{g} = [g_x, g_y, g_z]^T \) and \( g_z = -g \).
Using (1) and (2) the dynamics equations of the inverted 
pendulum can be derived as follows.
\[
p_x = \frac{m(c_z + g)\dot{c}_z - m\dot{c}_y c_z}{m(\dot{c}_z + g)} \tag{3}
\]
\[
p_y = \frac{m(c_z + g)\dot{c}_z - m\dot{c}_y c_z}{m(\dot{c}_z + g)} \tag{4}
\]
However (3) and (4) are non-linear. To attain linear 
equations the z-coordinate of the inverted pendulum 
is assumed to be constant. Let \( c_z = z_c \). Thus the (3) and (4) 
turn into linear equations as follows.
\[
p_x = c_x - \frac{1}{\omega^2_x} \ddot{c}_x \tag{5}
\]
\[
p_y = c_y - \frac{1}{\omega^2_y} \ddot{c}_y \tag{6}
\]
where \( \omega^2_x = \frac{g}{z_c} \). Henceforth, (5) and (6) are going to be 
referred as ZMP equations. Note that given the CoM 
coordinates of the pendulum \( C = [c_x, c_y, c_z]^T \) at any time it 
is straightforward to calculate the ZMP coordinates of the 
pendulum by (5) and (6). On the other hand walking 
trajectory generation is the inverse problem, in that, given 
a ZMP trajectory a COM trajectory should be found \[11\].
Thus, this trajectory of COM could be used as a reference 
for the COM of the actual biped walking robot. Further 
the legs should be in such coordination that this COM is 
tracked accurately. Since the goal is to achieve a 
dynamically stable gait the ZMP trajectory should always 
lie inside the supporting polygon. And this actually 
determines the location of the footprints of the biped 
robot. Finally by knowing the footprints and the COM 
trajectory by inverse kinematics relations a possible gait 
could be achieved.

B. Natural ZMP Trajectories

The ZMP for a walking robot can either be measured 
by means of force sensors or it can be computed. The 
ZMP of the robot should be always in the supporting 
polygon for it to be in a stable condition. This implies 
that the robot is continuously recovering from unbalanced 
conditions to a stable posture. Stable ZMP references can 
be employed to design stable walking patterns.

Usually in many reported studies \[11,15,16\], the ZMP 
reference in the single foot support phase is in the form of 
a point under the sole of the supporting foot. However, 
experiments with walking humans show that the ZMP 
does not stay at a fixed point in the single support phase, 
\[12-14\]. It rather passes the sole of the supporting foot, 
from the heel to the toe.

A natural ZMP trajectory during the human walk 
cycle is illustrated in Fig. 3. We believe that using natural 
ZMP reference trajectories for gait generation will result 
in a more natural and energy efficient CoM trajectory. In 
fact, already reported results also show that -since the 
resulting CoM trajectory oscillations are smoother- using 
variable ZMP trajectories result in more energy efficient 
trajectories \[14\].

C. Exact Solution of LIPM for Fixed ZMP

Recall the ZMP equations (5) and (6). Rearranging 
these equations,
\[
\ddot{c}_x = \omega^2_x c_x - \omega^2_x p_x \tag{7}
\]
\[
\ddot{c}_y = \omega^2_y c_y - \omega^2_y p_y \tag{8}
\]
From the equations (7) and (8) applying Laplace 
transform,
\[
C_x(s) = \frac{1}{1 - \frac{1}{\omega^2_x} s^2} \left[ p_x(s) - \frac{1}{\omega^2_x} C_x(0)s - \frac{1}{\omega^2_x} \dot{C}_x(0) \right] \tag{9}
\]
\[
C_y(s) = \frac{1}{1 - \frac{1}{\omega^2_y} s^2} \left[ p_y(s) - \frac{1}{\omega^2_y} C_y(0)s - \frac{1}{\omega^2_y} \dot{C}_y(0) \right] \tag{10}
\]
In (9-10) the following fixed ZMP trajectories are going 
to be used for the exact solution calculation. In Fig. 4 the 
\( x \)-axis (for sagittal plane) reference for ZMP trajectory, 
in Fig. 5 the \( y \)-axis (for frontal plane) reference for ZMP 
trajectory, and in Fig. 6 the resulting ZMP trajectory in the 
\( x - y \) plane can be seen. Note that Fig. 6 also 
indicates the foot placement positions in the \( x - y \) plane.

![Figure 3. A Natural ZMP Trajectory.](image)

![Figure 4. \( p_x^{ref} \), x-axis ZMP reference trajectory.](image)

![Figure 5. \( p_y^{ref} \), y-axis ZMP reference Trajectory.](image)
The ZMP reference trajectories in figures 4 and 5 can be expressed as follows.

\[
p^\text{ref}_x = B \sum_{k=1}^{\infty} \left( t - kT_0 \right) \quad (11) \\
p^\text{ref}_y = A \left( t \right) + 2A \sum_{k=1}^{\infty} (-1)^k \left( t - kT_0 \right) \quad (12)
\]

Applying Laplace transform to these equations and substituting them in (9) and (10) with zero initial conditions, the following equations can be derived.

\[
C_x(s) = \frac{1}{1 - \frac{1}{\omega_n^2} s^2} \left[ \frac{B}{s} e^{-\omega_n \sqrt{s}} + \frac{B}{s} e^{-2\omega_n \sqrt{s}} + \frac{B}{s} e^{-3\omega_n \sqrt{s}} + \ldots \right]
\]

\[
C_y(s) = \frac{1}{1 - \frac{1}{\omega_n^2} s^2} \left[ \frac{A}{s} e^{-\omega_n \sqrt{s}} + \frac{2A}{s} e^{-2\omega_n \sqrt{s}} - \frac{2A}{s} e^{-3\omega_n \sqrt{s}} + \ldots \right]
\]

With

\[
1 - \frac{1}{\omega_n^2} s^2 \frac{1}{s} = s - \left( s^2 - \omega_n^2 \right)^{-1}
\]

(13) and (14) can be rearranged to derive the following functions.

\[
C_x(s) = \left( \frac{1}{s} \frac{s}{s^2 - \omega_n^2} \right) e^{-\omega_n \sqrt{s}} + \frac{B}{s} \left( \frac{s}{s^2 - \omega_n^2} \right) e^{-2\omega_n \sqrt{s}} + \ldots
\]

\[
C_y(s) = \left( \frac{1}{s} \frac{s}{s^2 - \omega_n^2} \right) e^{-\omega_n \sqrt{s}} - 2 \frac{A}{s} \left( \frac{s}{s^2 - \omega_n^2} \right) e^{-\omega_n \sqrt{s}} + 2 A \left( \frac{s}{s^2 - \omega_n^2} \right) e^{-2\omega_n \sqrt{s}} - \ldots
\]

Finally, we can obtain the exact reference trajectories of the CoM by applying inverse Laplace transformations to equations (15) and (16).

\[
C_x(s) = B(1 - \cosh \omega_n (t - kT_0))(t - kT_0)
+ 2A(1 - \cosh \omega_n (t - 2kT_0))(t - 2kT_0) + \ldots
\]

\[
C_y(s) = A(1 - \cosh \omega_n (t - kT_0))
- 2A(1 - \cosh \omega_n (t - 2kT_0))(t - 2kT_0) + \ldots
\]

Although (17) and (18) are the exact solutions for the ordinary differential equations (5) and (6), in practice they are difficult to be used robustly for a real biped walking robot since they are composed of numerically unbounded \( \cosh(.) \) functions. Furthermore, they are unstable and very sensitive to the variation of \( \omega_n \). Therefore, an approximated solution composed of \( \sin(.) \) functions is suggested in [15] to serve as a robust CoM trajectory. This solution is outlined in the subsection below.

D. Planning an Approximate Solution

First an odd function with period \( T_0 \) is introduced from the x-directional reference ZMP \( p^\text{ref}_x \) of equation (11) as follows.

\[
p^\text{ref}_x(t) := p^\text{ref}_x(t) - \frac{B}{T_0} \left( t - \frac{T_0}{2} \right)
\]

\[
= \frac{B}{T_0} \left( t - \frac{T_0}{2} \right) \quad \text{and} \quad p^\text{ref}_x(t + T_0) = p^\text{ref}_x(t).
\]

Then assuming that the x-directional reference trajectory of CoM has the following form by using Fourier series,

\[
C^\text{ref}_x(t) = \frac{B}{T_0} \left( t - \frac{T_0}{2} \right) + \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{n\pi}{T_0} t \right) + b_n \sin \left( \frac{n\pi}{T_0} t \right) \right]
\]

Then applying the (20) to the ZMP differential equation (5) the following relation can be found.

\[
p^\text{ref}_x(t) = \frac{B}{T_0} \left( t - \frac{T_0}{2} \right) + \hat{p}^\text{ref}_x(t)
\]

where

\[
p^\text{ref}_x(t) = \sum_{n=2}^{\infty} \left[ a_n \left( 1 + \pi^2 \frac{T_0^2}{n^2} \right) \cos \left( \frac{n\pi}{T_0} t \right) + b_n \left( 1 + \frac{\pi^2}{4n^2} \right) \sin \left( \frac{n\pi}{T_0} t \right) \right]
\]

Here in the equation (21) the form of the odd function \( \hat{p}^\text{ref}_x(t) \) can be seen in Fig. 7. Since \( \hat{p}^\text{ref}_x(t) \) is an odd function with period \( T_0 \), the coefficients \( a_n = 0 \) and \( b_n \) can be found by solving the following equation.

\[
b_n \left( 1 + \frac{n^2\pi^2}{T_0^2} \frac{\omega_n^2}{\omega_n^2 + n^2\pi^2} \right) = \frac{2}{T_0} \int_{0}^{T_0} \hat{p}^\text{ref}_x(t) \sin \left( \frac{n\pi}{T_0} t \right) dt
\]

Finally \( b_n \) is found as

\[
b_n = \frac{BT_0^2 \omega_n^2 \left( 1 + \cos n\pi \right)}{n^2 \pi^2 \omega_n^2 + n^2 \pi^2}
\]

As a result, the x-directional reference trajectory of CoM can be obtained by substituting equation (24) to equation (20) as follows.

\[
C^\text{ref}_x(t) = \frac{B}{T_0} \left( t - \frac{T_0}{2} \right) + \sum_{n=1}^{\infty} \left[ \frac{BT_0^2 \omega_n^2 \left( 1 + \cos n\pi \right)}{n^2 \pi^2 \omega_n^2 + n^2 \pi^2} \right] \sin \left( \frac{n\pi}{T_0} t \right)
\]

On the other hand, since the y-directional reference ZMP \( p^\text{ref}_y(t) \) of the equation (12) is an odd function with
The y-directional reference can be found in a similar manner as follows.

\[
C^y_{ref}(t) = \sum_{n=0}^{\infty} \frac{2AT_0^2\omega_0^2(1-\cos n\pi)}{n\pi(T_0^2\omega_0^2 + n^2\pi^2)} \sin \left( \frac{n\pi}{T_0} t \right)
\]  

(26)

The resulting CoM trajectories for \( x \) and \( y \) axes can be seen in Fig. 8. and from Fig. 9.

E. Introduction of Natural ZMP References by Fourier Approximation to Obtain CoM Trajectories

As discussed in the previous sections the ZMP trajectory in a human walking cycle is not fixed at a point at certain periods but it travels under the supporting polygon. In the single support phase the ZMP travels from heel to the toe of the foot and in the double support phase it travels from the toe of the supporting foot to the heel of the swinging foot [12-14]. In this context the \( x \)-directional reference ZMP trajectory \( p^x_{ref} \) is introduced as follows (Fig. 10). Here \( b \) is the half length of the foot sole. It can be observed that in this trajectory ZMP travel starts from zero and advances in time under the sole of the foot in the initial single support phase and from heel to the toe of the foot in the further single support phases. By the same procedure followed in the previous sections we introduce the following odd function \( p'_x \) with period \( T_0 \) from the \( x \)-directional reference ZMP \( p^x_{ref} \), Fig. 11.

\[
p'_x(t) = p^x_{ref}(t) - \frac{B}{T_0} \left( t - \frac{T_0}{2} \right)
\]

(27)

Applying the same procedure from equation (19) to equation (23) we find the new \( b'_x \) coefficient as follows.

\[
b'_x = \frac{(B - 2b)T_0^2\omega_0^2(1+ \cos n\pi)}{n\pi(T_0^2\omega_0^2 + n^2\pi^2)}
\]  

(28)

Hence the natural CoM trajectory is found as

\[
C^x_{ref}(t) = \frac{B}{T_0} \left( t - \frac{T_0}{2} \right) + \sum_{n=0}^{\infty} \frac{(B - 2b)T_0^2\omega_0^2(1+ \cos n\pi)}{n\pi(T_0^2\omega_0^2 + n^2\pi^2)} \sin \left( \frac{n\pi}{T_0} t \right)
\]

(29)

The resulting \( C_x \) trajectory can be seen in Fig. 12. Note that this trajectory is smoother (shown in dashed line) than the conventional \( C_x \) trajectory (solid line) with fixed ZMP. The smoothness of the resulting trajectory implies that the acceleration differences are less when compared with the conventional \( C_x \) trajectory with fixed ZMP. This also implies that less energy is necessary to track the \( C_x \) trajectory with variable ZMP.
In this section the introduction of double support phases to previously used reference ZMP trajectories will be addressed. Adding double support phase to reference ZMP trajectories in both $x$ and $y$ axes by the method above, which is to blend lines with different slopes, makes it impossible to overcome such a problem. Instead the so-called Lanczos Sigma Factor is used for such a task.

The non-uniform convergence of the Fourier series for discontinuous functions is known as Gibbs Phenomenon in the literature. There are complex methods to smooth the Gibbs Phenomenon. One method is the so-called Lanczos Sigma Factor. In this approximation a function is multiplied by the coefficients in the Fourier partial sums. This function is a complex sine function involving the period of the original function. Fourier series by the Lanczos Sigma Factor can be rewritten as follows.

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \sin\frac{n\pi}{m} \left[ a_n \cos(n\theta) + b_n \sin(n\theta) \right]$$  

(30)

The resulting effect of the Lanczos Sigma Factor can be seen in Fig. 13 and Fig. 14.

In this example the Double Support Parameter $DSP$ of the Lanczos Sigma Factor $(\sin(c \frac{n\pi}{DSP}))$ is used to attain double support phases in the reference ZMP trajectory. Notice that in Fig. 14, the duration of the double support phase is tuned by setting appropriate values to the $DSP$ parameter. Also observe the variations of the CoM trajectory corresponding to different double support phase durations. Further the found Natural CoM trajectories for $x$–$y$ axes are as follows.

$$C_x^{\text{n}}(t) = \frac{B}{T_0} \left( t - \frac{T_s}{2} \right) + \sum_{n=1}^{\infty} \left[ (B-2b)BT_0^2\omega^2 \left[ 1 + \cos(n\pi) \right] \sin(c \frac{n\pi}{DSP}) \sin \left( \frac{n\pi}{T_0} \right) \right]$$  

(31)

$$C_y^{\text{n}}(t) = \sum_{n=1}^{\infty} \left[ 2AT_0^2\omega^2 \left[ 1 - \cos(n\pi) \right] \sin(c \frac{n\pi}{DSP}) \sin \left( \frac{n\pi}{T_0} \right) \right]$$  

(32)

In addition the Natural ZMP trajectories for $x$–$y$ axes are obtained by

$$p^{\text{n}}_{\text{base}}(t) = \frac{B}{T_0} \left( t - \frac{T_s}{2} \right) + \sum_{n=1}^{\infty} \left[ (B-2b)BT_0^2\omega^2 \left[ 1 + \cos(n\pi) \right] \sin(c \frac{n\pi}{DSP}) \sin \left( \frac{n\pi}{T_0} \right) \right]$$  

(33)

$$p^{\text{n}}_{\text{base}}(t) = \sum_{n=1}^{\infty} \left[ 2AT_0^2\omega^2 \left[ 1 - \cos(n\pi) \right] \sin(c \frac{n\pi}{DSP}) \sin \left( \frac{n\pi}{T_0} \right) \right]$$  

(34)

As an example, trajectory for the walking parameters close to a human’s is given in Fig. 15, and in Fig. 16. ($A=.15$[m], $B=.6$[m], $b=[.14]$ and $T_0 =1$[s])

IV. OUTLINE OF THE CONTROL ALGORITHM

The swing foot position references are obtained from ZMP and CoM references (Fig. 17). The control algorithm consists of five lower level position and force controller building blocks (Fig. 18). Swing foot references, or alternatively, the swing timing determines the timing for switching between control structures. However, swing reference timing is not the only criterion to switch from one control mode to the other. Switching from swing to support controller before actually reaching the ground level and establishing stable contact with the ground can cause a sudden loss of the robot balance. Therefore, ground interaction force information is used and controller mode switching is not allowed before the $z$-direction component of the contact force exceeds a certain threshold value. The force threshold value is a design parameter. The support-to-swing switching times are according to the swing timing without additional feedback from ground interaction forces. The double
support controller regards the biped robot as a trunk manipulated by two six-DOF arms with their bases positioned on the ground level (Fig. 19, left). The CoM position reference discussed above and fixed orientation reference with respect to the world coordinate frame are applied in a position control schemes for both manipulators. The position controllers running for the two manipulators (legs) are identical. Cartesian position and orientation errors are computed from the reference and actual position and orientations. These errors are reflected to the joint space errors by the use of inverse Jacobian relations. Independent joint PID controllers are employed for the joint space position control. The controllers for the two legs work almost independently. However, the Cartesian errors are scaled with different gains for the two legs before corresponding joint errors are computed. The scaling factor for the right leg is proportional to the horizontal distance of the left foot coordinate center from the CoM and similarly, the scaling factor for the left leg is proportional to the horizontal distance of the right foot from the CoM. This rule is obtained experimentally and it performed well for the coordination of the two legs in the double support phase.

The robot in swing phases can be seen as a ground based manipulator controlling the CoM position and trunk orientation and a second manipulator based at the hip controlling the swing foot position and orientation. The right support and left swing controllers are activated simultaneously. The single support controller applies the position control scheme described above for the double support phase (without using the scaling factors). The swing leg controller is a stiffness controller for the foot position and orientation. For soft landing purposes, a Cartesian stiffness matrix with low stiffness against in orientation errors and position errors in the z-direction is employed. The horizontal directions are penalized with higher stiffness coefficients.

V. SIMULATION RESULTS

Simulations studies are carried out with the robot model described in Section II, references generated in Section III and the coordination and control mechanism discussed in Section IV. The simulation scheme is similar to the one in [17]. The details of the algorithm and contact modeling can be found in [18]. Parameters used for reference generation are presented in Table II. Fig. 20 shows the y-direction CoM and CoM reference. It can be observed that the CoM reference in this direction is closely tracked except in the single support phases. The y-direction ZMP and ZMP reference curves displayed in Fig. 21 also a deviation from the reference curve in the swing phases. This suggests that the simple LIMP model, concentrating on the robot trunk, and ignoring the effects of the swing foot on the CoM of the whole robot, may encounter problems when the leg weight is not very low. The legs weigh 15 kg. Although it is much less than the 50 kg trunk weight, this weight affects the y-direction CoM and ZMP curves significantly. Apart from the swing phases, the tracking performance is quite acceptable. The x-direction CoM and ZMP curves together with their references are presented in figures 22 and 23, respectively. These curves, too, display oscillations and deviations from reference curves mainly due to the trunk dominated LIMP model.

Still, in the average, the reference curves are tracked.

<table>
<thead>
<tr>
<th>Table II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some of the important simulation parameters</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Step height</td>
</tr>
<tr>
<td>Step period</td>
</tr>
<tr>
<td>Foot to foot y-direction distance</td>
</tr>
<tr>
<td>Foot to foot y-direction ZMP reference distance</td>
</tr>
<tr>
<td>Ground interaction threshold force</td>
</tr>
</tbody>
</table>
In the average, the ZMP curve moves forward even in the single support phases. However, the transient behavior does not indicate that the naturalness of the human walk is achieved completely. Although there are some tracking problems as discussed above, the reference generation and control algorithms are generally successful, keeping the ZMP in the support polygon and enabling the robot move forward with an almost constant speed of 7 cm per second. This is achieved without the need for the elaborate trial and error steps common to many other reference generation approaches.

VI. CONCLUSION

A trajectory generation, coordination and control approach for biped walking robots is presented in this paper. Human-like ZMP reference trajectories with Fourier series approximation techniques for the solution of LIPM dynamics equations are employed in order to achieve naturalness in the walk. A control structure consisting of different modes and position and force control techniques is employed. Simulation studies show that the reference generation without considering the effects of the swing foot on robot ZMP can lead to significant deviations from reference trajectories. The walk, however, is stable and this is promising result making the algorithm a candidate for implementation.

REFERENCES


