1. Introduction

The control of a biped humanoid is a difficult task due to the hard-to-stabilize dynamics. Walking reference trajectory generation is a key problem. Reference generation techniques with the so-called Linear Inverted Pendulum Model (LIPM) are reported. Improved versions of the LIPM based reference generation are obtained by applying the Zero Moment Point (ZMP) Criterion, widely employed in the stability analysis of biped robot walk. Typically, the ZMP reference during a stepping motion is kept fixed in the middle of the supporting foot sole. This kind of reference generation lacks naturalness, in that, the ZMP in the human walk does not stay fixed, but it moves under the supporting foot. This paper proposes a reference generation algorithm based on single support foot ZMP references which move in directions parallel and perpendicular to the walking direction. A simple inverse kinematics based independent joint position controller is used in the full dynamics 3-D simulations with the model of a 12 Degrees of Freedom (DOF) biped robot. Simulation studies indicate that the moving ZMP references result in a more successful walk.

KEYWORDS
Humanoid robots, locomotion, Zero Moment Point

ABSTRACT
The control of a biped humanoid is a difficult task due to the hard-to-stabilize dynamics. Walking reference trajectory generation is a key problem. Reference generation techniques with the so-called Linear Inverted Pendulum Model (LIPM) are reported. Improved versions of the LIPM based reference generation are obtained by applying the Zero Moment Point (ZMP) Criterion, widely employed in the stability analysis of biped robot walk. Typically, the ZMP reference during a stepping motion is kept fixed in the middle of the supporting foot sole. This kind of reference generation lacks naturalness, in that, the ZMP in the human walk does not stay fixed, but it moves under the supporting foot. This paper proposes a reference generation algorithm based on single support foot ZMP references which move in directions parallel and perpendicular to the walking direction. A simple inverse kinematics based independent joint position controller is used in the full dynamics 3-D simulations with the model of a 12 Degrees of Freedom (DOF) biped robot. Simulation studies indicate that the moving ZMP references result in a more successful walk.
fixed and linear single support ZMP references too. A simple, independent joint position controller with position references obtained through inverse kinematics from the CoM reference is employed. Simulation results indicate that the walking performance with the new reference generation method is superior to the ones with fixed and linear ZMP reference trajectories. The Fourier series approximation based solution for the CoM for given reference ZMP trajectories is discussed in Section II. Section III outlines the joint control algorithm. Section IV is devoted to the simulation results and their analysis. Finally in Section V, we conclude in some remarks.

2. Reference Generation with Natural ZMP Trajectories

The sketch in Fig. 1 shows an example of a robot structure for which the reference generation and control algorithms presented below can be applied. The full dynamics description for a robot structure like the one shown in Fig. 1 is highly nonlinear, multiple dof and coupled. Closed form solutions for the dynamics parameters like the inertia matrix are very difficult to obtain. Typically, Newton-Euler recursive formulations are used in their computation [14]. Though obtaining the full dynamics model is very useful for the simulation and test of reference generation and control methods, a full dynamics model is too complex to serve as an intuitive model which can help in developing guidelines and basics for the walking control. Simpler models are more suitable for controller synthesis. The inverted pendulum model is such a simple model. The body is approximated by a point mass concentrated at the CoM of the robot. This point mass is linked to a stable contact point on the ground via a massless rod, which is the idealized model of a supporting leg.

![Fig. 1. Typical kinematic arrangement of a biped robot. It resembles an inverted pendulum in single support phases. The image on the right is a snapshot from the animation program used for the tests presented in Section IV.](image)

![Fig. 2. The Linear inverted pendulum.](image)

The swing leg is assumed to be massless too. Fig. 2 shows an inverted pendulum. In this figure, \( c = (c_x, c_y, c_z) \) stands for the coordinates of this point mass. The equations of motion of the inverted pendulum model are coupled and nonlinear. One more assumption, however, yields a linear system which is uncoupled in the \( x \) and \( y \)-directions. This is the assumption of fixed height of the CoM. This model is called LIPM and it is simple enough to work on and devise algorithms for reference generation [5]. Stability of the walk is trivially the most wanted feature of a reference trajectory. In biped robotics, the most widely accepted criterion for stability is based on the location of the ZMP [1]. For the arrangement in Fig. 2, the zero moment point is defined as the point on the \( x-y \) plane about which no horizontal torque components exist. The expressions for the ZMP coordinates \( p_x \) and \( p_y \) for the point mass structure in Fig. 2 are [13]

\[
p_x = c_x - \frac{z_c}{g} \ddot{c}_z \\

p_y = c_y - \frac{z_c}{g} \ddot{c}_y.
\]

In this equation, \( z_c \) is the height of the plane on which the motion of the point mass is constrained, \( g \) is the gravity constant (9.806 m/s\(^2\)). (1) and (2) are equations relating the ZMP and the CoM. For reference generation purposes a stable ZMP trajectory can be assigned without difficulty. The only constraint for stability of the robot is that the ZMP should always lie in the supporting polygon defined by the foot or feet touching the ground. The most intuitive choice for the ZMP location is the middle of the supporting foot sole. [13] created the reference ZMP trajectory shown in Fig. 4 with this idea. In this figure, \( A \) is the distance between the foot centers in the \( y \)-direction, \( B \) is the step size and \( T \) is half of the walking period. As can be observed from Fig. 4, firstly, step locations are determined. The staircase-like \( p_x \) and the square-wave structured \( p_y \) curves are fully defined by the selection of support foot locations if the half period \( T \) is given too.

However, the naturalness of the walk is not addressed in [13]. The starting point of the reference ZMP curves in that work is the choice that ZMP stays at a fixed point under the sole, though investigations [8,9] of the human ZMP revealed that it moves forward under the sole (Fig. 5). Further, the curves in Fig. 4 imply that the transition from left single support phase to the right support phase is instantaneous, without a double support phase.

[11] employs the \( p_x \) reference curve shown in Fig. 6.b. It can be noted that this figure illustrates a moving ZMP. It is more natural than a fixed ZMP curve because it is more similar to the human ZMP. The parameter \( b \) in the figure defines the range of the ZMP motion under the sole. A
symmetric trajectory centered at the foot center is assumed here. Though \( b \) can be interpreted as the half of the foot length, this interpretation is not a must: ZMP may move on the line connecting the heel with the toes without covering this line completely too.

Smooth transition between single support phases with a double support phase is achieved by an additional smoothing action discussed later in Section 4. This smoothing generates trapezoid shaped y direction ZMP references from the square wave shapes in Figures 4.c and 6.c.

It should, however, be noted that the shape of the ZMP curve shown in Fig. 5 is obtained from human walk with heel-first landing and toe-last take-off of the feet. In the human walk with feet parallel to the ground, the single support foot ZMP trajectories have the shape of an arc as shown in Fig. 7 [9]. The foot configuration parallel to ground is a choice which simplifies swing foot trajectory generation and it is widely used in gait synthesis. Further, the fixed y-direction ZMP references in Figures 4.c and 6.c (even after the above mentioned smoothing of the square wave shapes) present a discontinuity in the ZMP velocity reference. This discontinuity can lead to overshoots in tracking. Motivated by these facts, the y-direction ZMP reference trajectory in Fig. 8.c is proposed in this paper. This is a simple sinusoidal curve with period \( 2T \) and amplitude \( A+w \) where the \( w \) is a constant which should be chosen less than half of the foot width so that the reference ZMP remains under the single support foot.

Having defined the curves, and hence the mathematical functions for \( p^y_{\text{ref}}(t) \) and \( p^x_{\text{ref}}(t) \), the equations (1), (2) can be employed to obtain the solution for the CoM position references \( c^y_{\text{ref}}(t) \) and \( c^x_{\text{ref}}(t) \) (\( c^y_{\text{ref}}(t) = z_c \), which is a constant in the LIPM). After having found the required CoM reference trajectories, a position control scheme for the robot joints with references obtained by inverse kinematics from these center of mass locations can be obtained. Various control techniques can be applied for achieving desired CoM positions. In order to obtain CoM references which satisfy ZMP references in Figures 4, 6 and 8, Fourier series approximations can be employed. In [13], this method is used to generate CoM trajectories from the ZMP references in Fig. 4, [11] and [12] employ the technique in [13] for the ZMP curves in Fig. 6. In this paper, the same Fourier series approximation based is applied for the ZMP y-direction reference in Fig 8.c. The x-direction ZMP reference curves in Figures 6 and 8 are identical, and the Fourier series for the x-direction CoM reference is the same as the one presented in [12]:

\[
c^y_{\text{ref}} = \frac{B}{T} \left( t - \frac{T}{2} \right) + \frac{\alpha_y}{2} + \sum_{n=1}^{\infty} \alpha_n \cos\left( \frac{2\pi nt}{T} \right) + \beta_n \sin\left( \frac{2\pi nt}{T} \right)
\]

where

\[
\alpha_k = 0 \quad \text{for} \quad k = 0, 1, 2, 3, \cdots
\]

\[
\beta_k = \frac{(B - 2b)T^2}{k\pi(T^2 + k^2\pi^2)} \quad \text{for} \quad k = 1, 2, 3, \cdots
\]  

The infinite sum in (3) is approximated by a finite sum of \( N \) terms (\( N = 24 \) in the simulations in Section IV). The expression for the ZMP y -direction reference in Fig. 8 is

\[
p^y_{\text{ref}} = (A+w) \sin\left( \frac{2\pi mt}{2T} \right).
\]

Note the function in (5) is periodic with the period \( 2T \). It is reasonable to assume that \( c^y_{\text{ref}}(t) \) is a periodic function too and that it has the same period. Hence, it can be approximated by a Fourier series

\[
c^y_{\text{ref}}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos\left( \frac{2\pi kT}{T} \right) + b_k \sin\left( \frac{2\pi kT}{T} \right) \right).
\]

By (4) and (8), \( p^y_{\text{ref}} \) can be expressed as

\[
p^y_{\text{ref}}(t) = c^y_{\text{ref}} - \frac{z_c}{g} c^y_{\text{ref}}
\]

\[
= \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos\left( \frac{2\pi kT}{T} \right) + b_k \sin\left( \frac{2\pi kT}{T} \right) \right).
\]

Noting that this expression in the form of a Fourier series for \( p^y_{\text{ref}}(t) \), and since \( p^y_{\text{ref}}(t) \) is an odd function, we can conclude that the coefficients \( a_0 \) and \( a_k \) (\( k=1,2,3,\ldots \)) are zero. The coefficients \( b_k \) (\( k=1,2,3,\ldots \)) (and hence \( b_k \)) can be obtained by inspecting the Fourier series approximation for \( p^y_{\text{ref}} \):

\[
p^y_{\text{ref}} = (A+w) \sin\left( \frac{2\pi mt}{2T} \right) = \sum_{k=1}^{\infty} b_k \left( 1 + \frac{\pi^2 k^2}{\omega_k^2 T^2} \right) \sin\left( \frac{2\pi kT}{T} \right)
\]

\[
\Rightarrow b_k = \frac{(A+w)\omega_k^2 T^2}{\omega_k^2 T^2 + \pi^2}, \quad b_k = 0 \quad \forall \quad k > 1.
\]

This concludes the CoM reference generation section. The next section outlines the swing foot reference generation and locomotion control.

### 3. Outline of The Control Algorithm

The swing foot position references are obtained from ZMP and CoM references. The control algorithm is a simple one based on independent joint PID position controllers. The joint position references are generated through inverse kinematics from CoM and swing foot references defined in world frame coordinates. The foot orientation references used in inverse kinematics are fixed and they are computed for feet parallel to the robot body. The PID controller gains are obtained via trial and error. The controller structured this way, except for the joint PID controllers, is an open-loop one. However, it achieves walking when stable reference trajectories (like the ones obtained in the previous section) are employed.
4. Simulation Results

The biped model used in this paper consists of two 6-DOF legs and a trunk. Three joint axes are positioned at the hip. Two joints are at the ankle and one at the knee. Link dimensions of the biped are given in Table I. Simulations studies are carried out with this robot model, with the three types of references considered in Figures 4, 6 and 8. The introduction of the double support phase is achieved by a smoothing the ZMP reference curves as applied in [11] and [12]. Lanczos sigma factors [15] used to solve the Gibbs phenomenon [16], that is, non-uniform convergence of the Fourier series are employed for smoothing purposes. A view of the animation window is shown in Fig. 1. The details of the simulation algorithm and contact modeling can be found in [14]. Parameters used for reference generation are presented in Table II. Fig. 9 shows the CoM position and CoM reference position projection on the ground plane for a 11 step walk with fixed single support ZMP references (as in Fig. 4).
The COM reference is only roughly tracked. In the single support phases the deviation from the reference curve is more pronounced. The CoM and CoM reference on the ground plane with the forward moving single support ZMP references (as in Fig. 6) are shown in Fig. 10. Again, a steady walk is obtained and, the CoM trajectory tracking is improved when compared with the fixed ZMP reference case. CoM and CoM reference positions shown in Fig. 11 belong to the case with the arc shaped ZMP single support references (as in Fig. 8). The CoM reference is tracked more closely than with two previous cases. Our observation from the animations is that in the simulations with fixed ZMP references, the robot body inclines with larger angles at every step when compared with the forward moving and arc shaped ZMP reference cases. This is mainly due to higher cyclic acceleration and deceleration demands of the CoM reference (Fig. 9) obtained from the fixed ZMP reference curve. The use of the forward moving ZMP decreases the oscillations in body orientation significantly. The application of the arc shaped references further improves the walking by almost eliminating the oscillations. The body angles throughout the walk for the fixed ZMP, forward moving ZMP and arc shaped ZMP reference cases are displayed in Figures 12, 13 and 14, respectively. These figures indicate the improvement of the walking quality with the use of the arc shaped ZMP references too. Of special interest is the roll angle (angle of rotation about the world frame x-axis) comparison of the forward moving and arc shaped ZMP reference cases. This is because the introduction of the arc shape ZMP reference was motivated by our objective of smoothing the y-direction ZMP reference component for better control performance in this direction. Figures 15 and 16 show that the roll angle peak value drops by 40 % when the arc shaped references are employed. This justifies the proposed reference generation method. As a note on the controller performance it should be stated that a simple controller is employed in order to compare the contributions of different reference generation methods to the quality of the walk. In Fig. 11, where the best performance among the three cases is displayed with the arc shaped ZMP references, we can still observe that there are deviations from the desired CoM trajectory. This suggests that the LIMP model, concentrating on the robot trunk, and ignoring the effects of the swing foot on the CoM of the whole robot, may encounter problems when the leg weight is not very low. It can be deduced that techniques employing swing leg weight compensation can improve the walking of robots with heavy legs.

5. Conclusion

A trajectory generation, control approach for biped robots is presented. Arc shaped single support ZMP reference trajectories employed in order to improve walking performance. An independent joint position control structure is employed in the simulation studies. Walking performance with other types of ZMP references is tested too. The simulation results support the view that better walking performance can be achieved with human-like ZMP references.

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References


Table I: Masses and Dimensions of the Robot Links

<table>
<thead>
<tr>
<th>A. Link</th>
<th>Dimensions (LxWxH) [m]</th>
<th>Mass [kg]</th>
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<tr>
<td>Trunk</td>
<td>0.2 x 0.4 x 0.5</td>
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<td>Thigh</td>
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<td>Calf</td>
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<td>Foot</td>
<td>0.25 x 0.12 x 0.1</td>
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Table II: Simulation Properties

<table>
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<th>Parameter</th>
<th>Value</th>
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<td>Step period</td>
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<td>Foot to foot y-direction distance</td>
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<td>Step size</td>
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<td>ZMP motion under the support foot</td>
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