EGGS IN PG\((4n - 1, q), q\) EVEN, CONTAINING A
PSEUDO-CONIC

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ABSTRACT

An ovoid of PG\((3, q)\) can be defined as a set of \(q^2 + 1\) points with the property that every three points span a plane and at every point there is a unique tangent plane. In 2000 M. R. Brown ([7]) proved that if an ovoid of PG\((3, q), q\) even, contains a conic, then the ovoid is an elliptic quadric. Generalising the definition of an ovoid to a set of \((n - 1)\)-spaces of PG\((4n - 1, q)\) J. A. Thas [21] introduced the notion of pseudo-ovoids of eggs; a set of \(q^{2n} + 1\ (n - 1)\)-spaces in PG\((4n - 1, q)\), with the property that any three egg elements span a \((3n - 1)\)-space and at every egg element there is a unique tangent \((3n - 1)\)-space. We prove that an egg in PG\((4n - 1, q), q\) even, contains a pseudo-conic, that is, a pseudo-oval arising from a conic of PG\((2, q^n)\), if and only if the egg is classical, that is, arising from an elliptic quadric in PG\((3, q^n)\).

1. Introduction and preliminaries

An oval of PG\((2, q)\) is a set of \(q + 1\) points no three collinear. In 1954 it was shown by B. Segre [20] that if \(q\) is odd then an oval in PG\((2, q)\) is a conic. For \(q\) even, many ovals are known which are not conics (see [6] for a recent survey). An ovoid of PG\((3, q)\) is a set of \(q^2 + 1\) points such that every three points span a plane. If we exclude PG\((3, 2)\), that is, assuming \(q > 2\), then \(q^2 + 1\) is the maximal cardinality of a set of points satisfying this property. Moreover all the tangent lines to an ovoid at a certain point lie in a plane ([2], [17]); the tangent plane at that point. In 1955 A. Barlotti [2] and G. Panella [17] independently proved that an ovoid in PG\((3, q)\), \(q\) odd, is an elliptic quadric. For \(q\) even, one other example of an ovoid is known; called the Tits ovoid, which exists for \(q = 2^{2e + 1}, e \geq 1\). For results characterising the elliptic quadric and the Tits ovoid we refer to the survey [6]. A result fundamental to the proof of the main result of this paper is the following characterisation of the elliptic quadric ovoid.

**Theorem 1** (M. R. Brown [7]). Let \(\mathcal{O}\) be an ovoid of PG\((3, q), q\) even, and \(\pi\) a plane of PG\((3, q)\) such that \(\pi \cap \mathcal{O}\) is a conic. Then \(\mathcal{O}\) is an elliptic quadric.

An \((n - 1)\)-spread (partial \((n - 1)\)-spread) \(S\) of PG\((rn - 1, q)\) is a set of \((n - 1)\)-spaces such that any point of PG\((rn - 1, q)\) is contained in exactly (at most) one element of \(S\) (also called a spread if the dimension of the elements of \(S\) is understood). A spread \(S\) is called Desarguesian if the incidence geometry defined by taking the elements of \(S\) as points, the subspaces spanned by two different elements of \(S\) as

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lines, and the natural incidence relation (symmetric containment), is isomorphic to a Desarguesian projective space.

An egg \( E \) in \( \text{PG}(4n - 1, q) \) (or pseudo-ovoid) is a partial \((n - 1)\)-spread of size \( q^{2n} + 1 \), such that every three egg elements span a \((3n - 1)\)-space and for every egg element \( E \) there exists a \((3n - 1)\)-space \( T_E \) (called the tangent space of \( E \) at \( E \)) which contains \( E \) and is skew from the other egg elements. A pseudo-ovoid (or an egg in \( \text{PG}(3n - 1, q) \)) is a partial \((n - 1)\)-spread of size \( q^n + 1 \), such that every three elements of the pseudo-ovoid span \( \text{PG}(3n - 1, q) \). The notion of eggs was introduced by J. A. Thas in 1971 ([21]). An egg \( E \) in \( \text{PG}(4n - 1, q) \) is called a good egg if there exists an egg element \( E \) such that every \((3n - 1)\)-space containing \( E \) and two other egg elements contains exactly \( q^n + 1 \) egg elements. In that case \( E \) is called a good element of \( E \). If the elements of a pseudo-ovoid, respectively pseudo-oval, belong to a Desarguesian \((n - 1)\)-spread of \( \text{PG}(4n - 1, q) \), respectively \( \text{PG}(3n - 1, q) \), then the pseudo-ovoid, respectively pseudo-oval, is called elementary.

It follows that an elementary pseudo-oval arises from an oval of \( \text{PG}(2, q^n) \) and an elementary pseudo-ovoid arises from an ovoid of \( \text{PG}(3, q^n) \). If the oval is a conic we say that the elementary pseudo-oval is a pseudo-conic or a classical pseudo-oval, if the ovoid is an elliptic quadric then we call the pseudo-ovoid a classical pseudo-ovoid. In 1974 J. A. Thas proved that if every four egg elements span \( \text{PG}(4n - 1, q) \) or are contained in a \((3n - 1)\)-dimensional space, then the egg is elementary ([22]).

The only known examples of pseudo-ovals are elementary and pseudo-ovals have been classified by computer for \( q^n \leq 16 \) ([19]). More examples are known for pseudo-ovoids, all of them over a field of odd characteristic and they are connected to certain semifields (see Chapter 3 of [12] for a survey and [13] for recent results for the case when \( q \) is odd).

In this article we are concerned about pseudo-ovoids in the case when \( q \) is even. All known examples of eggs in \( \text{PG}(4n - 1, q) \), \( q \) even, are elementary. Pseudo-ovoids have been classified by computer for \( q^n \leq 4 \) ([14]). In 2002 J. A. Thas published the following two theorems.

**Theorem 2** (J. A. Thas [25]). An egg \( E \) of \( \text{PG}(4n - 1, q) \), with \( q \) even, is classical if and only if \( E \) is good at some element and contains at least one pseudo-conic.

**Theorem 3** (J. A. Thas [25]). An egg \( E \) of \( \text{PG}(4n - 1, q) \), with \( q \) even, is classical if and only if \( E \) contains at least two intersecting pseudo-conics.

In this article we prove that the only assumption one needs to conclude that an egg in \( \text{PG}(4n - 1, q) \), \( q \) even, is classical, is that it contains a pseudo-conic.

2. Eggs and translation generalized quadrangles

A (finite) generalized quadrangle (GQ) (see [18] for a comprehensive introduction) is an incidence structure \( S = (\mathcal{P}, \mathcal{B}, I) \) in which \( \mathcal{P} \) and \( \mathcal{B} \) are disjoint (non-empty) sets of objects called points and lines, respectively, and for which \( I \subseteq (\mathcal{P} \times \mathcal{B}) \cup (\mathcal{B} \times \mathcal{P}) \).
is a symmetric point-line incidence relation satisfying the following axioms:

(i) Each point is incident with $1 + t$ lines ($t \geq 1$) and two distinct points are incident with at most one line;

(ii) Each line is incident with $1 + s$ points ($s \geq 1$) and two distinct lines are incident with at most one point;

(iii) If $X$ is a point and $\ell$ is a line not incident with $X$, then there is a unique pair $(Y, m) \in \mathcal{P} \times \mathcal{B}$ for which $X I m I Y I \ell$.

The integers $s$ and $t$ are the parameters of the GQ and $\mathcal{S}$ is said to have order $(s, t)$. If $s = t$, then $\mathcal{S}$ is said to have order $s$. If $\mathcal{S}$ has order $(s, t)$, then it follows that $|\mathcal{P}| = (s + 1)(st + 1)$ and $|\mathcal{B}| = (t + 1)(st + 1)$ ([18, 1.2.1]). A subquadrangle $\mathcal{S}' = (\mathcal{P}', \mathcal{B}', \mathcal{I}')$ of $\mathcal{S}$ is a GQ such that $\mathcal{P}' \subseteq \mathcal{P}$, $\mathcal{B}' \subseteq \mathcal{B}$ and $\mathcal{I}'$ is the restriction of $\mathcal{I}$ to $(\mathcal{P}' \times \mathcal{B}') \cup (\mathcal{B}' \times \mathcal{P}')$. Let $\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ be a GQ of order $(s, t)$, $s \neq 1$, $t \neq 1$. A collineation $\theta$ of $\mathcal{S}$ is an elation about the point $P$ if $\theta = id$ or if $\theta$ fixes all lines incident with $P$ and fixes no point of $\mathcal{P} \setminus P'$. If there is a group $G$ of elations about $P$ acting regularly on $\mathcal{P} \setminus P'$, then we say that $\mathcal{S}$ is an elation generalized quadrangle (EGQ) with elation group $G$ and base point $P$. Briefly we say that $(\mathcal{S}^{(P)}', G)$ or $\mathcal{S}^{(P)}$ is an EGQ. If the group $G$ is abelian, then we say that the EGQ $(\mathcal{S}^{(P)}', G)$ is a translation generalized quadrangle (TQG) and $G$ is the translation group.

In PG$(2n + m - 1, q)$ consider a set $\mathcal{E}(n, m, q)$ of $q^m + 1$ $(n - 1)$-dimensional subspaces, every three of which generate a PG$(3n - 1, q)$ and such that each element $E$ of $\mathcal{E}(n, m, q)$ is contained in an $(n + m - 1)$-dimensional subspace $T_E$ having no point in common with any element of $\mathcal{E}(n, m, q) \setminus \{E\}$. It is easy to check that $T_E$ is uniquely determined for any element $E$ of $\mathcal{E}(n, m, q)$. The space $T_E$ is called the tangent space of $\mathcal{E}(n, m, q)$ at $E$. For $n = m = 1$ such a set $\mathcal{E}(1, 1, q)$ is an oval in PG$(2, q)$ and more generally for $n = m$ such a set $\mathcal{E}(n, n, q)$ is a pseudo-oval of PG$(3n - 1, q)$. For $m = 2n = 2$ such a set $\mathcal{E}(1, 2, q)$ is an ovoid of PG$(3, q)$ and more generally for $m = 2n$ such a set $\mathcal{E}(n, 2n, q)$ is a pseudo-ovoid. In general we call the sets $\mathcal{E}(n, m, q)$ eggs.

Now embed PG$(2n + m - 1, q)$ in a PG$(2n + m, q)$, and construct a point-line geometry $T(n, m, q)$ as follows. Points are of three types:

(i) the points of PG$(2n + m, q) \setminus$ PG$(2n + m - 1, q)$, called the affine points;

(ii) the $(n + m)$-dimensional subspaces of PG$(2n + m, q)$ which intersect PG$(2n + m - 1, q)$ in a tangent space of $\mathcal{E}(n, m, q)$;

(iii) the symbol $\infty$.

Lines are of two types:

(a) the $n$-dimensional subspaces of PG$(2n + m, q)$ which intersect PG$(2n + m - 1, q)$ in an element of $\mathcal{E}(n, m, q)$;

(b) the elements of $\mathcal{E}(n, m, q)$.

Incidence in $T(n, m, q)$ is defined as follows. A point of type (i) is incident only with lines of type (a); here the incidence is that of PG$(2n + m, q)$. A point of type (ii)
is incident with all lines of type (a) contained in it and with the unique element of $E(n, m, q)$ contained in it. The point $(\infty)$ is incident with no line of type (a) and with all lines of type (b).

**Theorem 4** (8.7.1 of Payne and Thas [18]). The incidence geometry $T(n, m, q)$ is a TGQ of order $(q^n, q^m)$ with base point $(\infty)$. Conversely, every TGQ is isomorphic to a $T(n, m, q)$. It follows that the theory of TGQ is equivalent to the theory of the sets $E(n, m, q)$.

In the case where $n = m = 1$ and $E(1, 1, q)$ is the oval $O$ the GQ $T(1, 1, q)$ is the Tits GQ $T_2(O)$. When $m = 2n = 2$ and $E(1, 2, q)$ is the ovoid $\Omega$, the GQ $T(1, 2, q)$ is the Tits GQ $T_3(\Omega)$. Note that $T_2(O) \cong Q(4, q)$ if and only if $O$ is a conic and non-classical otherwise, while $T_3(\Omega) \cong Q(5, q)$ if and only if $\Omega$ is an elliptic quadric (see [18, Chapter 3]). The kernel of $\mathcal{S} = T(n, m, q)$ is the maximum cardinality field $GF(q')$ for which there exists an $O(n', m', q')$ representing $\mathcal{S}$ and $\mathcal{S}$ may be represented by an $E(n'', m'', q'')$ if and only if $GF(q'') \subseteq GF(q)$ (see [18, Chapter 8]). Let $\mathcal{E}$ be an egg in $PG(4n - 1, q)$ and $T(\mathcal{E})$ the corresponding TGQ. If $O$ is a pseudo-oval of $E$ contained in $PG(3n - 1, q)$ and $PG(3n, q)$ any subspace containing $PG(3n - 1, q)$ not contained in $PG(4n - 1, q)$, then $PG(3n, q)$ induces a subquadrangle of $T(\mathcal{E})$ isomorphic to $T(O)$.

3. **Eggs in $PG(4n - 1, q)$, $q$ even, containing a pseudo-conic**

In this section we characterise the classical GQ $Q(5, q)$ as a TGQ with a single classical subquadrangle on the translation point. As a corollary we have the analogue of Theorem 1 for eggs.

We begin with a statement and sketch proof of an important lemma. The proof is a combination of results of [10], [23], [11], [24] and [15], and already noted in [25].

**Lemma 5.** Every $(2n - 1)$-dimensional space in $PG(3n - 1, q)$, $q$ even, skew from a pseudo-conic is the span of two elements of the Desarguesian spread induced by the pseudo-conic.

**Proof.** Let $U$ be a $(2n - 1)$-space skew from a pseudo-conic in $PG(3n - 1, q)$. Dualising in $PG(3n - 1, q)$ we obtain an $(n - 1)$-space $U'$ disjoint from a dual pseudo-conic, i.e. the set of $q^n + 1$ $(2n - 1)$-spaces corresponding to the $q^n + 1$ lines of a dual conic in $PG(2, q^n)$. By embedding $PG(2, q^n)$ in $PG(3, q^n)$ and dualising in $PG(3, q^n)$ one sees that the set of affine points of any $n$-space intersecting $PG(2, q^n)$ in $U'$ becomes a set of planes forming a semifield flock of a quadratic cone in $PG(3, q^n)$ and since $q$ is even the corresponding semifield is a field, which implies that $U$ corresponds to a line in $PG(2, q^n)$.

**Lemma 6.** Let $S$ be a TGQ of order $(s, s^2)$ with a translation point $(\infty)$ and a subquadrangle $S' = (\mathcal{P}', \mathcal{B}', \mathcal{I}')$ of order $s$ containing the point $(\infty)$. Then the egg corresponding to $S$ contains a pseudo-oval $O$ and $S'$ is a TGQ isomorphic to $T(O)$.

**Proof.** Suppose that the kernel of $S$ contains $GF(q)$ and $s = q^n$. Then let $\mathcal{E}$ be the corresponding egg in $PG(4n - 1, q)$ and represent $S$ as $T(\mathcal{E})$. The $q^n + 1$ lines of $S'$ incident with the point $(\infty)$ determine a set $\mathcal{O}$ of $q^n + 1$ egg elements.
$\{E_0, E_1, \ldots, E_{q^n}\}$. Let $A$ denote the set of affine points of $S'$. Let $Q \in A$ and consider the line $\langle E_0, Q \rangle$ in $S'$. It follows that every affine point of $\langle E_0, Q \rangle$ is contained in $A$. Let $P$ be an affine point in $\langle E_0, Q, E_1 \rangle \setminus \langle E_0, Q \rangle$. Then $\langle E_1, P \rangle$ intersects $\langle E_0, Q \rangle$ in an affine point $R \in A$, and hence $P \in A$. Hence all affine points of $\langle E_0, Q, E_1 \rangle$ are contained in $A$. Now consider any affine point $P$ in $\langle E_0, E_1, E_2, Q \rangle \setminus \langle E_0, Q, E_1 \rangle$. Then $\langle E_2, P \rangle$ intersects $\langle E_0, E_1, Q \rangle$ in a point $R \in A$. It follows that $A$ is the set of affine points of $\langle E_0, E_1, E_2, Q \rangle$ and $O$ is contained in $\langle E_0, E_1, E_2 \rangle$. This implies that $O$ is a pseudo-oval contained in $E$ and $S'$ is a TGQ isomorphic to $T(O)$.

**Theorem 7.** Let $S = \langle P, B, I \rangle$ be a TGQ of order $(s, s^2)$, $s$ even, with a translation point $(\infty)$ and a subquadrangle $S' = \langle P', B', I' \rangle$ isomorphic to $Q(4, s)$ containing $(\infty)$. Then $S \cong Q(5, s)$.

**Proof.** Suppose that the kernel of $S$ contains $GF(q)$ and $s = q^n$. Then let $E$ be the corresponding egg in $PG(4n - 1, q)$ and represent $S$ as $T(E)$. Now $S'$ is a (classical) subquadrangle of order $q^n$ containing $(\infty)$. By Lemma 6 $E$ contains a pseudo-conic in $PG(3n - 1, q)$ and $S'$ is constructed from a $PG(3n, q)$ containing $PG(3n - 1, q)$.

If $X$ is a point of $P \setminus P'$, then the lines incident with $X$ intersect $S'$ in a set $O_X$ of $q^{2n} + 1$ points of $S'$, no two collinear, called an ovoid of $S'$ ([18, 2.2.1]). The ovoid $O_X$ is said to be **subtended** by $X$. Suppose that $X$ is a point of type (ii) of $S'$, that is, a subspace of dimension $3n$ meeting $PG(4n - 1, q)$ in the tangent space at an egg element. Then $O_X$ consists of the point $(\infty)$ plus the $q^{2n}$ points $(X \cap PG(3n, q)) \setminus PG(3n - 1, q)$. The subspace $X \cap PG(3n - 1, q)$ is a $(2n - 1)$-dimensional subspace skew from the pseudo-conic $C$. From Lemma 5 we have that this is the span of two elements of the Desarguesian spread induced by the pseudo-conic. Representing $S'$ over $GF(q^n)$, that is, as $T_2(C)$ where $C$ is a conic in $PG(2, q^n)$, we see that $O_X$ consists of $(\infty)$ and the affine points of a plane of $PG(3n, q)$ skew from $C$. By the isomorphism from $Q(4, q^n)$ to $T_2(C)$ ([18]) it is clear that the ovoids of $T_2(C)$ consisting of $(\infty)$ and the affine points of a plane skew to $C$ correspond to the elliptic quadric ovoids of $Q(4, q^n)$ containing a fixed point. By a result of Bose and Shrikhande ([4]) any triad of $S$ has $q^n + 1$ centres and so a subtended ovoid of $S'$ may be subtended by at most two points of $S \setminus S'$, in which case the ovoid is said to be **doubly subtended**. Counting reveals that there are $q^{2n}(q^n - 1)/2$ elliptic quadric ovoids of $S'$ containing $(\infty)$ and $q^{2n}(q^n - 1)$ points of $P \setminus P'$ collinear with $(\infty)$ and hence subtending an ovoid of $S'$ containing $(\infty)$. Thus each such ovoid is doubly subtended.

Now let $Y$ be a point of $P \setminus P'$ not collinear with $(\infty)$ and $O_Y$ the ovoid it subtends in $S'$. We will consider this ovoid in the $T_2(C)$ model of $S'$. Since $Y \neq (\infty)$ it follows that $O_Y = A \cup \{p : P \in C\}$, where $A$ is a set of $q^{2n} - q^n$ affine points of $T_2(C)$ and $p$ is a point of type (ii) of $T_2(C)$ which is a plane containing $P \in C$. We now investigate the intersections of a plane $\pi$ of $PG(3, q^n)$ with $A$. If $p$ contains no point of $C$, then $\pi \cup (\infty)$ is an elliptic quadric subtended by two points, $X$ and $X'$ of $S \setminus S'$. If $Y$ is collinear with $X$ or $X'$, then $\pi \cap A$ is a single point. If $Y$ is not collinear with $X$ nor with $X'$, then $\{X, X', Y\}$ is a triad of $S$ and hence has $q^n + 1$ centres. Hence $|\pi \cap A| = q^n + 1$. Next suppose that $\pi$ contains a unique point $P = \pi_P \subset O_Y$, then $p$ contains no point of $A$. If $\pi \neq \pi_P$, then the $q^n$ lines of $\pi$ incident with $P$ and not in the plane of $C$ are lines of the $T_2(C)$ and so contain precisely one point of $A$. Hence $|\pi \cap A| = q^n$. Next suppose that $\pi$ contains
two points, $P$ and $Q$, of $C$. Of the $q^n + 1$ projective lines in $\pi$ incident with $P$ one is contained in $\pi_P$ and $q^n - 1$ are lines of $T_2(C)$ containing a unique point of $A$. Hence $|\pi \cap A| = q^n - 1$. Finally, if $\pi = \PG(2, q^n)$, then $\pi$ contains no point of $A$.

Consider the set of points of $\PG(3, q^n)$ defined by $\overline{O_Y} = A \cup C$. By the above the plane intersections with $\overline{O_Y}$ have size $1$ or $q^n + 1$ and a straightforward count shows that $\overline{O_Y}$ is an ovoid of $\PG(3, q^n)$.

Further, since $\overline{O_Y}$ contains the conic $C$ it is an elliptic quadric by Theorem 1. Hence the ovoid $\overline{O_Y}$ is an elliptic quadric ovoid of $\mathcal{S}'$ in the $Q(4, q^n)$ model. Thus we have that every ovoid of $\mathcal{S}' \cong Q(4, q^n)$ subtended by a point of $\mathcal{P} \setminus \mathcal{P}'$ is an elliptic quadric ovoid. By a theorem due independently to Brown ([8]) and Brouns, Thas and Van Maldeghem ([5]) it now follows that $\mathcal{S}$ is the classical GQ $Q(5, q^n)$.

\begin{remark}
In general, suppose that $\mathcal{S}$ is a TGQ of order $(s, s^2)$, $s$ even, represented by an egg $\mathcal{E}$ in $\PG(4n - 1, q)$. Suppose that $\mathcal{S}'$ is a subquadangle of $\mathcal{S}$ of order $s$, containing the base point ($\infty$) of $\mathcal{S}$. Then the argument at the start of the proof of Theorem 7 proves that $\mathcal{S}'$ is isomorphic to $T(O)$ for $O$ a pseudo-oval contained in $\mathcal{E}$. This solves an open case in [9].
\end{remark}

As a corollary we now have the main result of the paper.

\begin{theorem}
An egg $\mathcal{E}$ in $\PG(4n - 1, q)$, $q$ even, contains a pseudo-conic if and only if the egg is classical, that is arising from an elliptic quadric in $\PG(3, q^n)$.
\end{theorem}

\begin{proof}
Since an elliptic quadric contains conics, any egg arising from an elliptic quadric contains pseudo-conics. Now suppose $\mathcal{E}$ is an egg of $\PG(4n - 1, q)$ containing a pseudo-conic. Then $T(\mathcal{E})$ is a TGQ of order $(q^n, q^{2n})$ containing a classical subquadangle of order $q^n$ containing ($\infty$). By Theorem 7 $T(\mathcal{E})$ is the classical GQ $Q(5, q^n)$ and so by [1, Lemma 1] $\mathcal{E}$ arises from an elliptic quadric in $\PG(3, q^n)$.
\end{proof}

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\section*{References}

4. R. C. BOSCH AND S. S. SHREINER; Geometric and pseudo-geometric graphs ($q^2 + 1, q + 1, q$). J. Geom. 2/1 (1973) 75–94.
9. Matthew R. Brown, J. A. Thas; Subquadrangles of order s of generalized quadrangles of order (s, s^2). Part II. Submitted. J. Combin. Theory Ser. A.
13. Michel Lavrauw; Characterizations and properties of good eggs in PG(4n – 1, q), q odd. Submitted.
21. J. A. Thas; The m-dimensional projective space S_m(M_n(GF(q))) over the total matrix algebra M_n (GF(q)) of the n x n-matrices with elements in the Galois field GF(q). Rend. Mat. (6) 4 (1971), 499–532.
22. J. A. Thas; Geometric characterization of the [n – 1]-ovaloids of the projective space PG(4n – 1, q). Simon Stevin 47 (1973/74), 97–106.

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