

The Hilbert, Cohn-Vossen cubic surface

Anton Betten

Colorado State University

Cubic surfaces with 27 lines are a beautiful topic in geometry. In this talk, we discuss the problem of classifying cubic surfaces over finite and infinite fields. Classification up to isomorphism means determining the different orbits under the action of the symmetry group of projective space. Recent work on this problem over finite fields led to a renewed interest in a family of cubic surfaces with 27 lines over the real numbers with 6 Eckardt points. The surface is mentioned in Hilbert and Cohn-Vossen's book "Anschauliche Geometrie" from 1932. The surface is described in terms of a certain set of 12 lines in projective three-space. It is invariant under a group isomorphic to $\text{Sym}(4)$. There is hope that the methods of classification over finite fields might be able to draw attention to other interesting surfaces over the real numbers, where classification up to isomorphism is impossible. Further connections exist to the group of birational mappings of the plane, the Cremona group. Using the birational structure of surfaces, finite subgroups of the Cremona group arise.