Abstract

Polyominoes are, roughly speaking, plane figures obtained by joining squares of equal size (cells) edge to edge. We establish a connection of polyominoes to commutative algebra by assigning to each polyomino its ideal of inner minors (also called Polyomino ideals). This class of ideals widely generalizes the ideal of 2-minors of a matrix of indeterminates, and even that of the ideal of 2-minors of two-sided ladders. It also includes the meet-join ideal of plane distributive lattices. Typically one determines for such ideals their Gröbner bases, determines their resolution and computes their regularity, checks whether the rings defined by them are normal, Cohen-Macaulay or Gorenstein.

Let $\mathcal{P}$ be a collection of cells, $K$ be a field and $S$ be the polynomial ring over $K$ in the variables $x_a$ with $a \in V(\mathcal{P})$, where $V(\mathcal{P})$ is the vertex set of $\mathcal{P}$. We denote by $I_\mathcal{P} \subset S$ the ideal generated by the inner minors of $\mathcal{P}$ and by $K[\mathcal{P}]$ the quotient ring $S/I_\mathcal{P}$. We will investigate $I_\mathcal{P}$ for given different shapes of $\mathcal{P}$.