# ON THE ADDITIVE CYCLIC STRUCTURE OF QUASI-CYCLIC CODES

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Abstract. An index $\ell$, length $m\ell$ quasi-cyclic code can be viewed as a cyclic code of length $m$ over the field $\mathbb{F}_{q^\ell}$ via a basis of the extension $\mathbb{F}_{q^\ell}/\mathbb{F}_q$. However, this cyclic code is only linear over $\mathbb{F}_q$, making it an additive cyclic code, or an $\mathbb{F}_q$-linear cyclic code, over the alphabet $\mathbb{F}_{q^\ell}$. This approach was recently used in (Shi, Tang, Ge, Sok, Solé, 2017) to study a class of quasi-cyclic codes, and more importantly in (Shi, Wu, Solé, 2017) to settle a long-standing question on the asymptotic performance of cyclic codes. Here, we answer one of the problems posed in these two articles, and characterize those quasi-cyclic codes which have $\mathbb{F}_{q^\ell}$-linear cyclic images under a basis of the extension $\mathbb{F}_{q^\ell}/\mathbb{F}_q$. Our characterizations are based on the module structure of quasi-cyclic codes, as well as on their CRT decompositions into constituents. In the case of a polynomial basis, we characterize the constituents by using the theory of invariant subspaces of operators. We also observe that analogous results extend to the case of quasi-twisted codes.

*Keywords:* Quasi-cyclic code, additive cyclic code.