Combinatorial and geometric aspects of the group of projective motions of certain algebraic varieties

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In combinatorics and finite geometry, the study of algebraic groups and their various actions has often led to new constructions of interesting (rare) geometric objects. It is an essential feature of the interplay between groups and geometry. A well-known example, due to Jacques Tits from 1962, is the action of the Suzuki group on the points of a 3-dimensional projective space, giving rise to an ovoid (a notion introduced by Beniamino Segre): a set of points which has the same combinatorial and geometric properties as (but is not equivalent to) an elliptic quadric. Since then, this idea has matured, and the availability of computer algebra systems has greatly contributed to recent developments; there are many authors who have used so-called “orbit-stitching” to obtain new constructions of desirable (finite) geometries. In this talk we will focus on the action of the group of projective motions of certain algebraic varieties. The classification of their orbits on subspaces is a challenging task, and few classifications are complete. After a general introduction, I will explain some recent work, obtained jointly with Stefano Lia (University College Dublin) and Francesco Pavese (Politecnico di Bari), concerning the action of an algebraic group $G \leq \text{PGL}(4, q)$, isomorphic to $\text{PGL}(2, q)$, arising from a maximal rational curve embedded on a smooth Hermitian surface with some fascinating properties. The study of its orbits leads to a new construction of a quasi-Hermitian surface: a set of points with the same combinatorial and geometric properties as a non-degenerate Hermitian surface.