Dynamical Irreducibility of Pure Polynomials

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Let $f$ be a polynomial in $\mathbb{Q}[x]$. We say that $f$ is dynamically irreducible or stable over $\mathbb{Q}$ if all its iterates $f^n := f \circ f \circ \ldots \circ f$ are irreducible over $\mathbb{Q}$.

A polynomial defined over $\mathbb{Q}$ is said to be pure with respect to a prime $p$ if its Newton polygon consists of exactly one line, e.g., $p$-Eisenstein polynomials. In 1985, Odoni showed that Eisenstein polynomials are dynamically irreducible over $\mathbb{Q}$. Inspired by results of Odoni and others, we present new families of polynomials possessing a pure iterate which are dynamically irreducible over $\mathbb{Q}$. 