Additive Maximum Distance Separable Codes

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Let $A$ be a finite set and let $n$ and $k$ be a positive integers. An MDS code $C$ is a subset of $A^n$ of size $|A|^k$ in which any two elements of $C$ differ in at least $n - k + 1$ coordinates. In other words, the minimum (Hamming) distance $d$ between any two elements of $C$ is $n - k + 1$.

If $A$ is an abelian group then we define an additive code to be a code $C$ with the property that for all $u, v \in C$, we have $u + v \in C$. If $A$ is a finite field then $C$ is linear over some subfield of $A$, so we take $A = \mathbb{F}_{q^h}$ and assume that $C$ is linear over $\mathbb{F}_q$.

The trivial upper bound on the length $n$ of a $k$-dimensional additive MDS code over $\mathbb{F}_{q^h}$ is

$$n \leq q^h + k - 1.$$ 

The classical example of an MDS code is the Reed-Solomon code, which is the evaluation code of all polynomials of degree at most $k - 1$ over $\mathbb{F}_{q^h}$. The Reed-Solomon code is linear over $\mathbb{F}_{q^h}$ and has length $q^h + 1$.

The MDS conjecture states (excepting two specific cases) that an MDS code has length at most $q^h + 1$. In other words, there are no better MDS codes than the Reed-Solomon codes.

We use geometrical and computational techniques to classify all additive MDS codes over $\mathbb{F}_{q^h}$ for $q^h \in \{4, 8, 9\}$. We also classify the longest additive MDS codes over $\mathbb{F}_{16}$ which are linear over $\mathbb{F}_4$. These classifications not only verify the MDS conjecture for additive codes in these cases, but also confirm there are no additive non-linear MDS codes which perform as well as their linear counterparts.

In this talk I will cover the main geometrical theorem that allows us to obtain this classification and compare these classifications with the classifications of all MDS codes of alphabets of size at most 8, obtained previously by Alderson (2006), Kokkala, Krotov and Östergård (2015) and Kokkala and Östergård (2016).