Existence theorems for \( r \)-primitive elements in finite fields
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Abstract
Let \( r \mid q-1 \). An element of \( \mathbb{F}_q \) is \( r \)-primitive if it has order \((q-1)/r\). Thus, a primitive element is \( 1 \)-primitive and an \( r \)-primitive element is the \( r \)th power of a primitive element of \( \mathbb{F}_q \). We describe some existence theorems for general \( r \)-primitive elements and, in particular, analogues for \( 2 \)-primitive elements of the following complete existence theorems for primitive elements.

**Theorem A (1990).** For any \( n \geq 2 \) and \( a \in \mathbb{F}_q \) (necessarily with \( a \neq 0 \) if \( n = 2 \)) there exists a primitive \( \alpha \in \mathbb{F}_{q^n} \) with trace \( a \) over \( \mathbb{F}_q \), except when \( a = 0, n = 3, q = 4 \).

**Theorem B (1983).** Every line in \( \mathbb{F}_{q^2} \) contains a primitive element. (A line in \( \mathbb{F}_{q^2} \) is a set of the form \( \{ \beta(\gamma + a) : a \in \mathbb{F}_q \} \), for some nonzero \( \beta \in \mathbb{F}_{q^2}, \gamma \in \mathbb{F}_{q^2} \setminus \mathbb{F}_q \).)

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