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Bayes’ Theorem in the 21st Century

Bradley Efron

The term “controversial theorem” sounds like an oxymoron, but Bayes’ theorem has played this part for two-and-a-half centuries. Twice it has soared to scientific celebrity, twice it has crashed, and it is currently enjoying another boom. The theorem itself is a landmark of logical reasoning and the first serious triumph of statistical inference, yet it is still treated with suspicion by most statisticians. There are reasons to believe in the staying power of its current popularity, but also some signs of trouble ahead.

Here is a simple but genuine example of Bayes’ rule in action (see sidebar) (1). A physicist couple I know learned, from sonograms, that they were due to be parents of twin boys.

They wondered what the probability was that their twins would be identical rather than fraternal. There are two pieces of relevant evidence. One-third of twins are identical; on the other hand, identical twins are twice as likely to yield twin boy sonograms, because they are always same-sex, whereas the likelihood of fraternal twins being same-sex is 1/2. Thus, the odds of the twins being identical are 2:1. Using Bayes’ rule, the probability of identical twins given the sonogram is 1/3 divided by the odds of the sonogram, or about 17%. Hence, the odds of the twins being identical rather than fraternal are 2:1.

In the 21st century, Bayes’ rule plays an increasingly prominent role in statistical applications but remains controversial among statisticians.
can be done legitimately has fueled the 250-year controversy.

Frequentism, the dominant statistical paradigm over the past hundred years, rejects the use of uninformative priors, and in fact does away with prior distributions entirely. In place of past experience, frequentism considers future behavior. An optimal estimator is one that performs best in hypothetical repetitions of the current experiment. The resulting gain in scientific objectivity has carried the day, though at a price in the coherent integration of evidence from different sources, as in the FiveThirtyEight example.

The Bayesian-frequentist argument, unlike most philosophical disputes, has immediate practical consequences. Consider that after a 7-year trial on human subjects, a research team announces that drug A has proved better than drug B at the 0.05 significance level. Asked why the trial took so long, the team leader replies “That was the first time the results reached the 0.05 level.” Food and Drug Administration (FDA) regulators reject the team’s submission, on the frequentist grounds that interim tests of the data, by taking repeated samples, could raise the false alarm rate to (say) 15% from the claimed 5%.

A Bayesian FDA regulator would be more forgiving. Starting from a given prior distribution, the Bayesian posterior probability of drug A’s superiority depends only on its final evaluation, not whether there might have been earlier decisions. This is a corollary of Bayes’ theorem, convenient but potentially dangerous in practice, especially when using prior distributions not firmly grounded in the coherent integration of evidence from different sources, as in the FiveThirtyEight example.

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SIDEBAR: BAYES’ THEOREM IN ACTION

If \( P(A) \) is the probability of A and \( P(B) \) is the probability of B, then the conditional probability of A given B is \( P(A|B) \) and the conditional probability of B given A is \( P(B|A) \). Bayes’ theorem says that

\[
P(A|B) = \frac{P(B|A)P(A)}{P(B)}
\]

In the twins example, A is “twins identical” and B is “sonogram shows twin boys.” The doctor’s prior says \( P(A) = 1/3 \); genetics implies \( P(B|A) = 1/2 \) and \( P(B|not \ A) = 1/4 \), so \( P(B) = (1/2)(1/3) + (1/4)(2/3) = 1/3 \). Bayes’ theorem then gives

\[
P(A|B) = (1/2)(1/3)/(1/3) = 1/2
\]

The two pieces of evidence thus balance out, and the likelihood of the boys being fraternal is equal to that of the boys being identical.

References and Notes
4. D. Singh et al., Cancer Cell 1, 203 (2002).

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