If the volatility of stock market prices is to be understood in terms of the efficient markets hypothesis, then there should be evidence that true investment value changes through time sufficiently to justify the price changes. Three indicators of change in true investment value of the aggregate stock market in the United States from 1871 to 1986 are considered: changes in dividends, in real interest rates, and in a direct measure of intertemporal marginal rates of substitution. Although there are some ambiguities in interpreting the evidence, dividend changes appear to contribute very little toward justifying the observed historical volatility of stock prices. The other indicators contribute some, but still most of the volatility of stock market prices appears unexplained.

Why are stock market prices so volatile? The standard deviation from 1871 to 1986 of the January-to-January percentage change in the real Standard and Poor composite stock price index $P_t$ (Fig. 1, solid line) is 17%. The real index rose 85% between 1927 and 1929, and fell 52% between 1929 and 1932. It rose 69% between 1954 and 1957. It fell 56% between 1973 and 1975. What is it that is so different about the demand for, or supply of, corporate shares from one year to the next that might account for such big price movements?

Price changes have long been attributed to psychological factors: investor overreaction to earnings, dividends, or other news, waves of social optimism or pessimism, fashions or fads. However, statistical evidence amassed during the past few decades has been widely interpreted as implying that markets are “efficient” (1, 2).

According to the efficient markets hypothesis, price changes occur when new information about the true investment value of stocks becomes available to the public: the price changes are big because the information is about something very important. However, statistical analyses (3–9) of aggregate historical data have recently raised questions of whether something sufficiently large does indeed happen to true investment value to justify the price movements.

The Simple Efficient Markets Model

The simple efficient markets model states that the real return $R_t$ on stocks cannot be forecasted, that all information about future prices is efficiently incorporated in today’s price, and one can never know that stocks are a better or worse investment today than at any other time. Formally, the model can be written:

$$E[R_t] = r$$

$$R_t = (P_{t+1} - P_t + D_t)/P_t$$

Here, $R_t$ denotes the total real return (capital gain plus dividend income) from holding stocks between the time $t$ and time $t + 1$ as a proportion of the initial investment $P_t$, $D_t$ denotes the real dividends paid on the stock between time $t$ and $t + 1$, and $r$ is a constant. $E_t$ denotes the mathematical expectation (or optimal forecast) conditional on the set of information available to the public at time $t$, a set which includes $P_{t-1}, P_{t-2}, \ldots, D_{t-1}, D_{t-2}, \ldots$ and current and lagged values of other data. Equation 1 may be described as a random walk hypothesis for prices corrected for dividend payments.

It follows (10) from Eq. 1 (and a condition that prices do not have an upward trend through time if dividends do not) that price is given by

$$P_t = E_t P_t'$$

$$P_t' = D_t/(1 + r) + D_{t+1}/(1 + r)^2 + D_{t+2}/(1 + r)^3 + \ldots$$

That Eq. 3 also implies Eq. 1 can be easily verified by substitution into Eq. 2 (11). In words, $P_t$ is the optimal forecast of $P_t'$, the true investment value, and $P_t'$ is the present value, discounted at constant rate $r$, of actual future real dividends. $P_t'$ may also be described as the perfect-foresight price, which is the price that would obtain by the efficient markets model (Eqs. 1 or 3) if everyone knew all future dividends with certainty.

The Variability of Forecasts and Forecasted Variables

The optimal forecast of any random variable $x$ cannot be as volatile as $x$ itself unless the forecast is very accurate. If, for example, the forecast of $x$ were as volatile as $x$ but only weakly correlated with it, then high values of the forecast would tend to be associated with negative forecast errors, low values with positive forecast errors. This would then mean that the forecast error was somewhat forecastable, and so the forecast could not be optimal.

Formally, the optimal forecast $E_x$ must satisfy

$$\sigma(E,x) = \rho(E,x, x)\sigma(x)$$

where $\rho$ denotes the correlation coefficient and $\sigma$ the standard deviation. Since $\rho$ cannot exceed 1.00, in no cases can the volatility of the optimal forecast exceed that of the variable forecasted.

The Variability of Prices and Perfect-Foresight Prices

One can use Eq. 5 to test the model Eq. 3 by comparing $P_t'$ and comparing this with the actual real price $P_t$. This was done (Fig. 1, dotted line) with the actual real dividend series $D_t$, which is shown in Fig. 2. The discount rate $r$ was taken as the average real return $R_t$ for this index over the entire sample, which was 8.2% (with a standard error of ±1.6%). Of course no one now knows annual dividends after 1985; it was assumed that these are such as to make

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Fig. 1. Real stock prices and their perfect-foresight counterparts, 1871 to 1986 (4, 9). $P_t$ (solid line) is the Standard and Poor composite stock price index in January of year $t$ times 100 and divided by $\pi_t$, the producer price index for year $t$. The series $p_t$ is for January starting in 1900, annual average before that $P_t$ (dotted line) is computed from Eq. 4 in the text using the dividend series $D_t$ that is shown in Fig. 2 $P_t^*$ (dashed line) is computed from Eq. 7 in the text using $r_t = \pi_t[\pi_t + (1 - r_t/200) (1 - r_{t+1}/200)]^{-1} - 1$, where $r_{t+1}$ is the January value and $r_{t+2}$ is the July value of the prime 4- to 6-month commercial paper rate (6-month rate starting in 1979) in annual percent.

$P'_{1986} = P_{1986}$. Changing this assumption for $P'_{1986}$ would have only the effect of dragging up or down the recent values of $P_t'$ [note that 1/(1.082)^{10} = 0.45, so that the weight the assumption has in determining $P'_{1973}$ is less than half that in determining $P'_{1986}$, that in determining $P'_{1968}$ is only 4% of that in determining $P'_{1986}$].

As can be seen from Fig. 1, the $P_t'$ so computed is dramatically less variable than the actual real price $P_t$. The forecast $P_t$ has shown tremendous variability, whereas the variable forecasted, $P_t'$, has remained quite close to a smooth trend line. This appears to contradict Eq. 5 where $\sigma(\pi)$ is the standard deviation of detrended $P_t'$ (defined as the ratio of $P_t'$ to an exponential curve, the long-run deterministic trent)," drawn through $P_t'$) and $\sigma(\pi(\pi))$ is the standard deviation of detrended $P_t'$, (defined as the ratio of $P_t'$ to the same smooth curve) Thus finding and analogous findings in terms of other measures of variability have been interpreted as seriously calling into question the simple efficient markets model (3-9).

One must be cautious in concluding from this evidence that stock prices are too volatile to satisfy Eq. 3. Since the half-life of the weighting pattern defining $P_t'$ in Eq. 4 is about a decade, then even though we have 116 annual observations, we essentially have no more than about a dozen more or less independent observations of $P_t'$ We should not regard the year-to-year choppiness of $P_t$ when compared to $P_t'$ as evidence against the model Eq. 1 (12). Still, a half dozen observations could suffice to reject a statistical hypothesis convincingly, if the evidence is strong enough. Moreover, although the efficient markets model does not necessarily imply this, it might have turned out that people have a great deal of information about future dividends, which would imply substantial correlation even between the year-to-year changes in $P_t$ and $P_t'$, that would be convincing evidence for the efficient markets model, which was not found.

Criticisms

The conclusion that the behavior of $P_t'$ is inconsistent with the efficient markets model Eq. 3 has been quite controversial. One concern of critics (13) has related to the fact that with "smooth" time series, the sample standard deviation is in small samples a downward biased measure of the true standard deviation. Since $P_t'$ is a smoothed version of dividends, there is reason to suspect that the downward bias might be greater for the estimated standard deviation of $P_t'$ than it is for the estimated standard deviation of $P_t$. If $P_t'$ were only a little more variable than $P_t$, then we might attribute the apparent violation of Eq. 5 to such bias, and not to the failure of the efficient markets model. However, Kledon (14) argued that this bias alone is unlikely to account for the fact that the standard deviation of detrended $P_t'$ is so much higher than that of $P_t'$.

Another criticism (15, 16) of the conclusion against market efficiency is that it relies on the assumption that the trend path of dividends was known in advance: the so-called "deterministic trend" may in fact be a random walk and the uncertainty about future dividends may well have been much larger than is suggested by the slight variations around the ultimate realization of the random walk. This criticism is potentially important but not conclusive (17), and several papers have provided alternative tests that sought to deal with this criticism, tests that were interpreted as showing excess volatility relative to Eq. 3 for stock prices (7-9).

Price-Dividend Ratios

A simple way to respond to the criticism concerning trend is to detrend by dividing by $D_{t-1}$, and compare $P_t/D_{t-1}$ with $P_t'/D_{t-1}$. Since $D_{t-1}$ is known at time $t$, the detrending at time $t$ makes use only of information available at that time. Equation 3 implies that $P_t/D_{t-1}$ equals the optimal forecast of $P_t'/D_{t-1}$, and $P_t'/D_{t-1}$ is proportional to a weighted average over $k$ of $D_{t+1-k}$. The model thus says that $P_t/D_{t-1}$ should be high when dividends can be forecasted to increase in the not-too-distant future, and low when dividends can be forecasted to decrease.

A plot of $P_t/D_{t-1}$ and $P_t'/D_{t-1}$ appears in Fig. 3. Here, $P_t/D_{t-1}$ is more variable than $P_t'/D_{t-1}$, but it is only slightly more variable. However, since the correlation $\rho$ between $P_t'/D_{t-1}$ and $P_t/D_{t-1}$ is only 0.03, we may say using Eq. 5 with $s = P_t'/D_{t-1}$ that $P_t/D_{t-1}$ is vastly more variable than it should be given the information it conveys about future dividend changes. It is worth noting, though, that $P_t'/D_{t-1}$ and $P_t'/D_{t-1}$ (in Fig. 1 share some movements, each over a few years or so, until the early 1950s. (The correlation between the two series 1872 to 1950 is 0.06.) These shared movements might be regarded as suggesting some element of truth to the efficient markets model (9). The shared movements occur because some of the transient short-run movements in dividends are not fully reflected in price, and both $P_t/D_{t-1}$ and $P_t'/D_{t-1}$ share the same...
denominator. But this is faint praise for the efficient markets model. If \( P_t \) had instead been equal each year just to a simple sum of real dividends over the preceding 20 years, implying \( P_t \) had been much less volatile than it was, then the correlation for 1891 to 1986 between \( P_t/D_{t-1} \) and \( P^*_t/D_{t-1} \) would have been 0.72.

**An Alternative Efficient Markets Model with Interest Rate Data**

The evidence regarding market efficiency does not take into account any expected variability of real interest rates. To those accustomed to hearing that the market rally since 1982 is due to declining interest rates, this omission will seem important. Equation 1 may be replaced by 

\[
E_t(R_t - r_t) = g,
\]

where \( r_t \) is the 1-year real interest rate (the nominal interest rate corrected for the actual inflation between time \( t \) and time \( t + 1) \), and \( g \) is a constant. This alternative acknowledges that stocks earn more on average than do interest-bearing assets, presumably to compensate stockholders for the greater risk inherent in stocks, but that the relative attractiveness of stocks versus interest-bearing assets does not change through time. It follows as above then (disregarding the fact that the 1-year real interest rate \( r_t \) is not known with certainty at time \( t \)) that

\[
P_t = E_tP^*_t
\]

\[
P^*_t = D_t/(1 + r_t + g) + D_{t+1}/[(1 + r_t + g)(1 + r_{t+1} + g)] + D_{t+2}/[(1 + r_t + g)(1 + r_{t+1} + g)(1 + r_{t+2} + g)] + \ldots
\]

\([7]\)

\( P_t \) is the optimal forecast of \( P^*_t \), the true investment value in this model, and \( P^*_t \) is just the present value of future dividends discounted by the actual future interest rates.

As above, \( P^*_t \) was computed with data on actual dividends paid and also data on real interest rates, the latter computed with historical data on prime commercial paper. The expected return differential \( g \) between stocks and commercial paper was estimated as the sample mean differential from 1871 to 1985: 4.8% with a standard error of \( \pm 1.7\% \). The \( P^*_t \) with this \( g \) and with a terminal condition that makes \( P^*_t/100 = P_{1000} \) is plotted in Fig. 1 (dashed line). \( P^*_t \) is slightly less variable than \( P_t \), contrary to the implications of the model, and shows virtually no correlation with \( P_t \) except for trend. The stock market rally from 1982 to 1986 is indeed explained by the change in real interest rates, but other movements in \( P_t \) are not matched by movements in \( P^*_t \). There is a conspicuous shared movement in \( P^*_t \) and \( P_t \) between the years 1920 and 1929, but a sharply opposite movement between 1929 and 1933. \( P_t/D_{t-1} \) and \( P^*_t/D_{t-1} \) (Fig. 5) are negatively correlated (1872 to 1986, \( \rho = -0.02 \)).

**A Consumption-Based Efficient Markets Model**

Another way to incorporate time-varying real discount rates into the efficient markets hypothesis is to look at nonfinancial evidence that might suggest movements in rates of discount. Per capita real-consumption expenditure at time \( t \), \( C_t \), is an indicator of current economic well-being for individual investors. When \( C_t \) is low relative to expected future \( C_t \), it is plausible that demand for stocks would be low, and thus expected return (and hence the rate at which future dividends will be discounted to today’s price) must be high to induce investors to hold existing shares. In simple terms, in bad times people will be inclined to sell some of the shares to consume the proceeds, and so price must fall to clear the market. For this purpose, bad times must be defined as times when people expect higher \( C_t \) in the future; they would not be tempted to consume their savings if they expected worse times yet in the future.

This intuitive notion may be formalized by specifying that the representative investor maximizes an expected utility function

\[
E_tU_t = E_t[u(C_{t+1})/(1 + h) + u(C_{t+2})/(1 + h)^2 + u(C_{t+3})/(1 + h)^3 + \ldots \] \( u(C_{t+k}) \) is the instantaneous utility from consuming \( C_{t+k} \), \( k \geq 0 \), and \( h \) is the subjective rate of time discount. If the representative consumer chooses consumption subject to a budget constraint to maximize this utility function, it follows that the Euler equation

\[
u'(C_t) = E_t[(1 + R_t)\nu'(C_{t+1})]/(1 + h) \] \( \nu \) is the marginal utility, that is, the derivative of the instantaneous utility function. The left-hand side of the Euler equation is the rate at which marginal utility is gained by consuming the proceeds of the investment the following period. If the former did not equal the latter, the individual would be better off either by saving more or consuming more. It follows from the Euler equation as above that \( P_t \) is given by

\[
P_t = E_tP^*_t
\]

\[
P^*_t = D_tS_{t+1} + D_{t+1}S_{t+2} + D_{t+2}S_{t+3} + \ldots
\]

\[S_{t+k} \equiv u'(C_{t+k})/[\nu'(C_t)(1 + h)^k]
\]

Here, \( P^*_t \) is the true investment value and \( S_{t+k} \) is the intertemporal marginal rate of substitution between \( C_t \) and \( C_{t+k} \).

We can compare \( P^*_t \) using historical data on \( C_t \) and \( D_t \), if we assume a functional form for \( u(C_t) \); a common form in the theoretical finance literature is \( u(C_t) = C_t^{-\phi}/(1 - \phi) \), \( A > 0 \). The larger the parameter \( A \) (the coefficient of relative risk aversion), the more the representative investor is assumed to dislike variation in consumption, and the more volatile the \( P^*_t \) will be. With \( \phi = 4 \), and \( 1 + h \) chosen as the sample mean for 1889 through 1984 of \( (1 + R_t)(C_tC_{t+1})^4 \), \( P^*_t \) is about as volatile as \( P_t \). (Fig. 4) (6)

For the earlier part of the sample there is some positive correlation in \( P^*_t \) and \( P_t \) beyond trend, although there is essentially no correlation in the years since 1950. The correlation we have discovered here is essentially the well-known correlation of the stock...
market with the so-called "business cycle," a correlation that has diminished since 1950 (6).

For large enough $A$, (considerably above 4) we will find that Eq. 5 is satisfied where $x$ is detrended $P_t$, Grossman, Mehra, and Shiller (19) estimated $A = 21.2$ for the sample 1890 to 1983. Such a high value of $A$ may be implausible. For this estimated $A$, the Euler equation implies, for example, that a person who knows that consumption $C_t$ will increase 5% between this year and next will require an expected return on stocks of over 180% in order to be willing to hold the existing shares.

Anyway, movements in the aggregate stock market are largely independent of movements in prices of other long-term assets such as housing or land (20). This is not what we would expect if these rate of discount movements dominated the movements in the stock market.

Statistical Issues

Changes in stock market prices have a probability density that differs distinctly from the usual bell-shaped normal density. The density is "fat-tailed," meaning that occasional big outlier observations tend to occur, outliers that are many sample standard deviations from the mean. The stock market tends to bring up occasional new "surprises" that seem outside the expected range provided by conventional statistical methods. The fact that the usual normal density is not really well suited for statistical analysis poses serious problems for formal hypothesis testing with financial data. Mandelbrot (21) has suggested that the normality assumption be replaced by the assumption of another distribution in the stable Pareto family. Such distributions have infinite standard deviations. The sample standard deviation will of course always be finite, but in no way can it be regarded as an estimate of the true standard deviation. In this case, one cannot use Eq. 5, which was the foundation of the above analysis.

Our understanding of the fat-tailed nature of changes in stock prices has been altered by the discovery that the variability of changes in stock prices changes through time and that this variability can be forecasted. For example, Bollerslev (22) has shown in the context of a "generalized auto-regressive conditional heteroskedasticity model" of time-varying variability that the distribution of changes in the monthly postwar Standard and Poor composite index conditional on recent changes is somewhat less fat-tailed than the unconditional distribution. In his model, the unconditional probability density of stock price changes does have a finite standard deviation.

The problem posed by the fat-tailed density is however just one manifestation of a deeper statistical problem. There is no well-defined theoretical statistical model of true investment value, and thus there is no way to be sure that any measures of historical variability accurately represent the potential variability. It is conceivable, for example, that the stock market's ups and downs during the last century represented genuine new information about a potential big disaster (for example, a nationalization) or big windfall (for example, a technological revolution) that would utterly change the outlook for future dividends. There is really no way to ensure that people were not right to change their minds from time to time about the possibility of such a rare big event. All we can say is that nothing actually happened in history that would seem to justify the price movements. Those who criticize the studies finding excess volatility in stock prices under the assumption that dividends are a random walk (14) or that managers smooth dividends (15) are really also relying on an assumption that the outlook for future dividends is much different than the historical record would suggest; their criticisms imply that dividends will not always stay close to a simple trend line in the future as they have in the past.

Conclusion

The price $P_t$ or its ratio to dividend $P_t/D_{t-1}$ generally appears to show too much variability given its correlation with its perfect-foresight counterpart under any of the models considered here. This is not to say that there might not be an element of truth to some of the models (witness some of the shared movements in subperiods of the series noted above), and statistical significance of these results is a difficult issue still unresolved in the literature. But there is certainly little indication that the source of stock price movements ought to be considered explained by any of these models. For the aggregate stock market, the widespread impression that there is strong evidence for market efficiency may be due just to a lack of appreciation of the low power of many statistical tests (23). It should be borne in mind, however, that there are individual investment assets whose true investment value does not look like a simple trend, for some of these, true investment value may predictably change sharply, even by orders of magnitude. For such assets, the efficient markets hypothesis does appear to suggest useful models. The notion of efficient markets, of course, also has value in the simple sense that stock market returns are not highly forecastable.

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