(1) HW1 is from Papadimitrou & Lewis: 4.1.4, 4.1.7, 4.1.10, 4.2.1, 4.2.2; and is due 27.2.2012

(2) HW2 is due 5.3.2012
   (Q1) Describe in detail the multi-tape TMs that implement the following 3 instructions of RATM:
   (a) add = c
   (b) jump s
   (c) sub j
   (Q2) Papadimitrou & Lewis: 4.3.2, 4.3.4, 4.3.6, 4.4.2

(3) HW3 is due 19.3.2012

(Q1) Let $\Sigma$ be a finite alphabet set.
(a) Prove that $\Sigma^*$ is a countable set
(b) Prove that the set of all subsets of $\Sigma^*$ is not a countable set
(c) Prove that the set of all finite subsets of $\Sigma^*$ is a countable set

(Q2) Show that the functions below are $\mu$-recursive.
(a) $gcd(n,m)$, {greatest common divider of $n$ and $m$}
(b) $lcm(n,m)$, {lowest common multiple of $n$ and $m$}
(c) prime (n) {nth prime number}
(d) $g(k,n) = \mu m \{ (k+2)^m > n+1 \}$
(e) $f(n) = g(n,n)$, {g as in part (d)}
(f) $h(k) = \sum_{j=0, P} g(k,j)$, {g as in part (d) and $P$ is a constant integer}

(Q3) If $f : \mathbb{N} \rightarrow \mathbb{N}$ is both $\mu$-recursive and invertible (i.e. bijective: 1-to-1 and onto) prove that $f^{-1}$ is also $\mu$-recursive

(Q4) Prove that the function next $(n,k)$ defined in class is primitive recursive
(4) HW4 is due 26.3.2012

(Q1)
(a) Give the full description of the 2-counter machine that simulates a stack machine. In particular carefully specify the internal states and the transitions of the simulating counter machine.
(b) Repeat part (a) for the 2-counter machine simulation of 4 counter machines.

(Q2) For the Universal Turing Machine $U$ described in class:
(a) If originally the data (input) is given as: $\text{blank} <M> \text{blank} <\omega>$ in the first tape where originally the head points at the first blank and tape 2 and tape 3 are clean with the heads at the leftmost blank slot describe the initialization TM component of $U$ that will put this into the standard 3-tape setting as described in class. Here the notation $<M>$ and $<\omega>$ stands for the encoded versions of the TM and its input respectively. More precisely for $E$ denoting the symbol encoding function:

$$< M > = E(s) , E(q_1) , E(\sigma_1) , E(q_1') , E(\text{act}_1) \ldots (E(q_n) , E(\sigma_n) , E(q_n') , E(\text{act}_n))$$

with $s$ as the initial state and

$$< \omega > = E(u_1) , E(u_2) , \ldots , E(u_k)$$

where $\omega = u_1 u_2 \ldots u_n$ and the usual convention of blip blank $\omega$ configuration on Tape 1 is used with the head on the blank.
(b) Modify the 8 sub-Turing Machines that make-up $U$ so that it halts at a special halt state, say $\text{halt}_w$ when the encoding is false.

(Q3) Specify a new Universal Turing Machine, say $V$, by modifying $U$ given in class where the symbol set of $V$ is given by: $\Sigma_V = \{ \text{blank}, ',', ',', 1 \}$

(Q4) Describe verbally, but carefully and in detail, a Universal Random Access Turing Machine that can simulate any given RATM.

(Q5) Write down the initial configuration of $U$ after your encoding of a TM:
(i) that cleans up a given tape filled with $0$s and $1$s
(ii) that interchanges $0$s and $1$s in the input