THEORETICAL & EXPERIMENTAL INVESTIGATION OF COMPOSITE STRUCTURES

Melih Papila
Research on Composite Materials and Applications

DESIGN OPTIMIZATION

SURROGATE MODELS
DESIGN OF EXPERIMENTS

SENSITIVITY TO UNCERTAINTY

ANALYSIS

TESTING
Overview

Optimization of Cracked Composite Panels...

Analysis
Design Optimization
Given stacking sequence, find ply thicknesses

Piezoresistive microphone design...

Piezoelectric Composite Plate...

ANALYSIS & DESIGN OPTIMIZATION

Buckling & Post-buckling of Composite...

ANALYSIS & TESTING

Tailoring wing structures...

Given stacking sequence, find ply thicknesses

\[ K \leq K_0 \]

\[ \sigma_2 = \frac{K}{\sqrt{2\pi}} \]

Minimized Detectable Pressure

Compromise design and least sensitive to uncertainty

Built-in twist compensation for structural deformation at the design condition

At an off-design condition, fixed compensation does not provide optimal load distribution for the minimum drag

Such a drag penalty may be eliminated by tailoring structural deformation as the flight condition changes – Composite wing
Theoretical & Experimental Investigation of Composite Structures

- Buckling and Post-buckling of Composite I-Sections

- Objective
  - Develop an efficient and accurate analysis tool to predict Buckling load and Post-buckling capacity of Composite I-sections


Implementation of Composite Panel Stress Fracture Constraint in a Commercial FE based Structural Optimization Code

Given stacking sequence, find ply thicknesses

\[ K \leq K_Q \]

\[ \sigma_y = \frac{K}{\sqrt{2\pi r}} \]

Piezoresistive microphone design: Tradeoff between sensitivity and noise floor

Minimized Detectable Pressure:
Compromise design and least sensitive to uncertainty

Axisymmetric Piezoelectric Composite Plate Configurations for Optimum Volume Displacement

Oscillating Piezo-Composite Diaphragm

Orifice

Cavity

Net Flow

Unimorph | Bimorph

Bimorph/unimorph

Amount of PZT 1.05
Natural frequency 1.00
Volume displacement 1.42

Tailoring wing structures for reduced drag penalty in off-design flight conditions

- Built-in twist compensates for structural deformation at the design condition
- At an off-design condition, fixed compensation does not provide optimal load distribution for the minimum drag
  - 2% penalty
- Such a penalty may be significant in lifetime of airplane.
- Can be eliminated by tailoring structural deformation as the flight condition changes – Composite wings

Overview

Optimization of Cracked Composite Panels...

Piezoresistive microphone design...

TESTING ANALYSIS DESIGN OPTIMIZATION

Piezoelectric Composite Plate...

Tailoring wing structures...

Buckling & Post-buckling of Composite I-Sections
THEORETICAL & EXPERIMENTAL INVESTIGATION OF COMPOSITE STRUCTURES

Buckling and Post-buckling of Composite I-Sections
Acknowledgement

- **Prof. Mehmet A. Akgun**, Dept. of Aerospace Engineering – METU

- **Dr. Necip Pehlivanturk**, Roketsan Missile Industries

- **Assoc. Prof. Peter Ifju and Dr. Xiaokai Niu**, Experimental Stress Analysis Laboratory - Mechanical and Aerospace Eng. Dept. University of Florida

Outline

- **Motivation & Objective**
- **Analysis – Buckling and Post-buckling**
  - Governing equations
  - Assumptions
  - Solution method
  - Verification
- **Testing**
- **Comparison of theoretical and experimental work**
- **Concluding Remarks**
Thin-walled Columns, Why Composite?

- Where the strength/weight ratio matters – Aerospace Engineering
- Where corrosion resistance needed – Civil Engineering
Composite I-section, Why Buckling?

Behavior under compressive loading usually a design constraint...

- Given
  - Dimensions: length, web and flange widths
  - Material & Stacking sequence
- Find
  - Load to carry without buckling

How much weight can be saved with composite I-section compared to Aluminum under the same compressive load?
Composite I-Section, Why Post-buckling?

- **Given**
  - Dimensions: length, web and flange widths
  - Material & Stacking sequence

- **Find**
  - Load to carry without buckling
  - Can Carry loads after buckling

Further weight and material savings?
Solving for Buckling and Post-buckling Issues?

Commercial packages available, e.g., STAGS

- Cumbersome when number of analyses are needed
  - Design Optimization
  - Uncertainty analysis
  - Reliability based design optimization
- Difficult to integrate with an optimizer
Objective

Develop an efficient and accurate analysis tool to predict Buckling load and Post-buckling capacity of Composite I-sections.
Outline

- Motivation & Objective
- Analysis – Buckling and Post-buckling
  - Governing equations
  - Boundary conditions & Assumptions
  - Solution method
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- Comparison of theoretical and experimental work
- Concluding Remarks
A thin walled structural member such as an I-section consists of a number of plate elements.

- The buckling problem for a thin walled section is treated as a plate problem.
- Governing equation of laminated composite plates.
Composite I-section Plate Elements

Stack of fiber reinforced plies

Stacking sequence: $\theta_1 / \theta_2 / \cdots / \theta_N$

Specially orthotropic laminates: $[0/90]_s = 0/90/90/0$
Buckling of Composite I-Section Column

Equation governing specially orthotropic laminated plate elements

\[ D_{11} w_{,xxx} + 2 (D_{12} + 2D_{66}) w_{,xyy} + D_{22} w_{,yyy} + N_x w_{,xx} = 0 \]

where

\[ w \text{ : lateral deflection} \]

\[ [D] \text{ : bending stiffness matrix} \]

\[ N_x \text{ : in-plane stress resultant} \]
Buckling of Composite I-Section Column

- For thin walled structures made of plate elements, governing equation is written for each plate separately.

- Exact solution exists if strain distribution is uniform (*perfect plate and columns*), Lee (1986), Papila (1995).

- Approximate method is needed if strain distribution is no longer uniform (*imperfect column or post-buckling regime*).
Buckling of Composite I-Section Column Boundary Conditions

- Both exact and approximate method need to incorporate boundary conditions
  - Continuity conditions at the web and flange junctions
  - Free edge conditions
  - Support at the loading ends (simply-supported)
Buckling of Composite I-Section Column Assumptions

- A short strut of an I-section with simply supported ends buckles in such a way that
  - The angle between adjacent plate elements along their junction is preserved
  - Number of half waves, $m$
  - Common longitudinal edges of plate elements remain straight
Buckling of Composite I-Section Column
Galerkin’s Method

- Assumed deflection, \( w \) as weighted sum of polynomial functions compatible with the boundary conditions and assumptions

\[
\begin{align*}
    w^B(x, y) &= \text{Galerkin Integral in Equilibrium equation} \\
    \mathbf{P}_{BuC} &= \min \{ P_{cr}^m \}
\end{align*}
\]

\[
\begin{bmatrix} G_1 \end{bmatrix} \{ C \} - P_{cr}^m [G_2] \{ C \} = 0
\]

\([G_1], [G_2] : \text{Material, stacking sequence, integrals of assumed functions}
\{C\} : \text{Eigenvector, weights for assumed functions}

\( P_{cr}^m \) : Smallest Eigenvector, Critical or buckling load for \( m \) many half waves
Buckling of Composite I-Section Column Verification

Comparison for isotropic material: Exact vs Galerkin

Characterizing web-flange junction pays off!
Buckling of Composite I-Section Column Verification

Column

$[(0/90/0/90)_S]$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Present (N)</th>
<th>STAGS (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>7813</td>
<td>7918</td>
</tr>
<tr>
<td>Mode 2</td>
<td>8316</td>
<td>8446</td>
</tr>
<tr>
<td>Mode 3</td>
<td>10119</td>
<td>10294</td>
</tr>
<tr>
<td>Mode 4</td>
<td>10464</td>
<td>10538</td>
</tr>
</tbody>
</table>
Outline

- Motivation & Objective
- **Analysis – Buckling and Post-buckling**
  - Governing equations
  - Boundary Conditions & Assumptions
  - Solution method
  - Verification
- Testing
- Comparison of theoretical and experimental work
- Concluding Remarks
Post-buckling of Composite I-Section Column

Equilibrium equation

\[
D_{11} w_{,xxxx} + 2(D_{12} + 2D_{66}) w_{,xyy} + D_{22} w_{,yyyy} \\
- N_x w_{,xx} - 2 N_{xy} w_{,xy} - N_y w_{,yy} = 0
\]

Compatibility equation (von Karman)

\[
a_{22} \frac{\partial^4 F}{\partial x^4} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} + a_{11} \frac{\partial^4 F}{\partial y^4} = w_{,xy}^2 - w_{,xx} w_{,yy}
\]
Post-buckling of Composite I-Section Column Assumptions

- Assumptions in initial buckling analysis apply
  - The angle between adjacent plate elements along their junction is preserved
  - Common longitudinal edges of plate elements remain straight

- The lateral deformation in the post-buckling regime is assumed to be a magnification of the buckling mode shape, \( \Omega w^B(x,y) \)
  - The number of half waves \( m \) for the buckling remains the same
  - Critical eigenvector remains the same
Post-buckling of Composite I-Section Column
Galerkin’s Method

Lateral deflection
\[ w^P = \Omega \times w^B (x, y) \]
Stress function
\[ F(x, y) \]
Compatiblity
\[ \{b\} = \Omega^2 [M][CC] \]
Equilibrium
\[ PI_1 + \Omega^2 I_2 + [R]\{b\} = 0 \]

\{b\} : Coefficient vector for stress function

\[ [M], [R] : \text{Material, stacking sequence, BCs} \]

\{C\} : Eigenvector from buckling analysis

\[ P \] : Load in post-buckling regime, \( P \geq P_{Buc} \)

\[ I_1, I_2 : \text{Material, stacking sequence, BCs} \]
Post-buckling of Composite I-Section Column Verification by STAGS

Slope is the post-buckling stiffness, $k_{post}$

Couple of seconds

Versus

Kernel time of 3 minutes
Summary-ANALYSIS

- Given material, dimensions and stacking sequence
  - Initial local buckling load
  - Post-buckling stiffness
- Verification by commercial tool STAGS
- Potential benefits
  - Can be used for further weight reduction as pointed out in motivation
  - Fast results, ideal in design optimization, uncertainty evaluations
- Need to validate assumptions holding for both STAGS and present analysis:
  - Magnification of the buckling critical mode and
  - No mode change in post-buckling regime
Outline

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- Testing
  - Manufacturing of I-sections
  - Test-setup
- Comparison of theoretical and experimental work
- Concluding Remarks
Cytec

T800 / 5245C
Carbon fiber / Epoxy

30 mm wide Unidirectional fiber reinforced pre-preg tape

D 0/90/0/90/90/0/90/0
E 0_2 / 90 / 0 / 90 / 0 / 90 / 0_2
Manufacturing
Molding the I-Section Column
Manufacturing Curing in Autoclave

Used autoclave at Roketsan

<table>
<thead>
<tr>
<th></th>
<th>Cure Temperature</th>
<th>Cure Pressure</th>
<th>Cure time</th>
</tr>
</thead>
<tbody>
<tr>
<td>T800/5245 C</td>
<td>177° C</td>
<td>7 bars</td>
<td>135 minutes</td>
</tr>
</tbody>
</table>

http://www.cerritos.edu/ctc/images
Trimming and Tab Installation

Tabs to distribute the loading uniformly and to prevent premature failure due to contact stresses.
Testing- Overview

- Material: T800/5245 UD tape pre-preg
- Two different stacking sequence, D and E
- Total of 13 I-section columns tested at
  - Roketsan (8 columns), Setting I
  - Experimental Stress Analysis Lab of Mechanical and Aerospace Eng. Dept., University of Florida (5 columns), Setting II
Test Set-up at Roketsan
Setting I

- INSTRON universal testing machine
- Video tape recorder
- Dial gages
Test Set-up at UF Setting II

- INSTRON universal testing machine
- Video tape recorder
- LVDT
- Strain Gages
- Shadow Moire
Outline

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Testing Buckling Load via Strain Gages

Specimen E5

- Strain gage 1
- Strain gage 2
Buckling Load via Averaging Strain Gage Readings

\[
\frac{1}{2} \left( \text{Gage 2} + \text{Gage 1} \right)
\]

Specimen E5

Load, \( P \) (N)

Average Strain %

-0.125
-0.1
-0.075
-0.05
-0.025
0
0.025
0.05
0.075
0

5000 10000 15000 20000 25000

-0.08826

8800

Data below \( P_{cr} \)

Data above \( P_{cr} \)
Results - Buckling Load

Column D

23 mm
39 mm
170 mm

Predicted Buckling Load $P_{Buc}$ (N)

<table>
<thead>
<tr>
<th>Column</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>D8</td>
<td>6651</td>
</tr>
<tr>
<td>D9</td>
<td>7220</td>
</tr>
<tr>
<td>D10</td>
<td>6363</td>
</tr>
</tbody>
</table>

Average $P_{Buc}$ (N) 6745

STAGS (clamped ends) $P_{Buc}$ (N) 7025

About 8% underprediction

Mismatch of end conditions
### Results - Buckling Load

<table>
<thead>
<tr>
<th>Column</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>7464</td>
</tr>
<tr>
<td>E2</td>
<td>9242</td>
</tr>
<tr>
<td>E5</td>
<td>8800</td>
</tr>
</tbody>
</table>

- **Predicted Buckling Load**
  - \( P_{Buc} \) (N)
  - Average: 8502
  - STAGS (clamped ends): 9341

- **About %10 underprediction**
- **Mismatch of end conditions**
Post-buckling Stiffness via Dial Gage/LVDT Reading

Specimen D2

Load, $P$ (N)

Square of maximum web deflection, $w^2$ (mm$^2$)

- dial gage measurement
- linear fit for measurement
- theoretical
Post-buckling Stiffness via Dial Gage/LVDT Reading

\[
P(N) = 1000 \times \sqrt{w^2 (mm^2)}
\]

![Graph showing the relationship between Load, \(P\), and the square of the maximum web deflection, \(w^2\). The graph includes a linear fit for measurement and a theoretical curve.](image)

Specimen E2
Results – Post-Buckling Stiffness

Column D

23 mm
39 mm
170 mm

Predicted Post-Buckling stiffness $k_{post}$ (N/mm$^2$)

<table>
<thead>
<tr>
<th>Column</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>D8</td>
<td>1517</td>
</tr>
<tr>
<td>D9</td>
<td>1455</td>
</tr>
<tr>
<td>D10</td>
<td>1603</td>
</tr>
</tbody>
</table>

Average $k_{post}$ (N/mm$^2$) 1525

About %8
Results – Post-Buckling Stiffness

Predicted Post-Buckling stiffness $k_{\text{post}}$ (N/mm$^2$)

<table>
<thead>
<tr>
<th>Column</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>1469</td>
</tr>
<tr>
<td>E2</td>
<td>1983</td>
</tr>
<tr>
<td>E5</td>
<td>1721</td>
</tr>
</tbody>
</table>

Average $k_{\text{post}}$ (N/mm$^2$) 1724

Average percentage error (12 columns): 15%

About %27
Summary-TESTING

- Analysis helped making the instrumentation decisions
- Tests validated assumptions holding for both STAGS and present analysis:
  - magnification of the buckling critical mode and
  - no mode change in post-buckling regime
- Buckling Load predicted within 10%
- Post-buckling stiffness predicted within 15%
I-Section Design with Buckling Constraint
Composite versus Aluminum

- Given
  - Dimensions: length, web and flange widths
  - Material & Stacking sequence
- Find
  - Load to carry without buckling

How much weight can be saved with composite I-section compared to Aluminum?

<table>
<thead>
<tr>
<th>Property</th>
<th>Ratio (T800/5245)/Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buckling load</td>
<td>1</td>
</tr>
<tr>
<td>Thickness</td>
<td>1.29</td>
</tr>
<tr>
<td>Density</td>
<td>0.59</td>
</tr>
<tr>
<td>Weight</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Saving About %25
I-Section Design Alternative Based on Post-buckling Behavior

Given
- Dimensions: length, web and flange widths
- Material and Stacking sequence

Find
- Load to carry without buckling
- Can Carry loads after buckling

![Graph showing Load vs. Square of maximum web deflection]

Allows further weight and material savings

\[ \frac{W_D}{W_E} = 0.89 \]
Outline

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  - Solution method
  - Verification
- Testing
  - Manufacturing of specimens
  - Test-setup
- Comparison of theoretical and experimental work
- Concluding Remarks
Concluding Remarks

Analysis & Testing, complement each other

- Analysis and prediction helped for instrumentation decisions such as gage locations.
- Experiments validated key assumptions
  - No mode shape change
  - Common edges remain straight
Concluding Remarks…

Objective:
Develop an analysis tool, efficient and accurate in predicting Buckling load and Post-buckling capacity of Composite I-sections

- A code developed for prediction of buckling load and post-buckling capability of I-Sectons
- Efficient: matter of seconds to determine both
- Accurate:
  - verified by a commercial code STAGS (within 1%)
  - verified by testing (within 15%)

- Advantageous for
  Design Optimization and Uncertainty Analysis