# THE DAMPED HARMONIC OSCILLATOR

**GOAL:** This experiment starts with the simple harmonic oscillator. The effects of adding first a damping force will be investigated.

**INTRODUCTION:** The simple harmonic oscillator is one of the central problems in physics. It is useful in understanding springs, small amplitude pendulums, electronic circuits, quantum mechanics, and even cars that shake at 53 MPH. Furthermore, many problems can be considered the sum of a large number, or infinite number, of harmonic oscillators.

Almost everyone has an intuitive understanding of the playground swing, and so it is a good first example. If the person in the swing is neither “pumping” nor being pushed, and if frictional losses are small, one has a simple harmonic oscillator, at least for small amplitudes. If the rider drags his or her feet then there is *damping.*

(Actually, in normal use the swing’s amplitude is too large for the motion to be that of a simple harmonic motion. Dragging one’s feet does not produce exactly the “right” kind of friction, and the force from the person pushing is not sinusoidal. But it is still a good example, for developing intuitively.)

The gory mathematics will be saved for the section on theory. But take a moment to look at eq. 7 which is the final working equation for this experiment. To use this equation, one will need to measure a fair number of quantities. It will be easier to make some of these measurements if one sets experimental controls, that is, one simplifies the experiment.

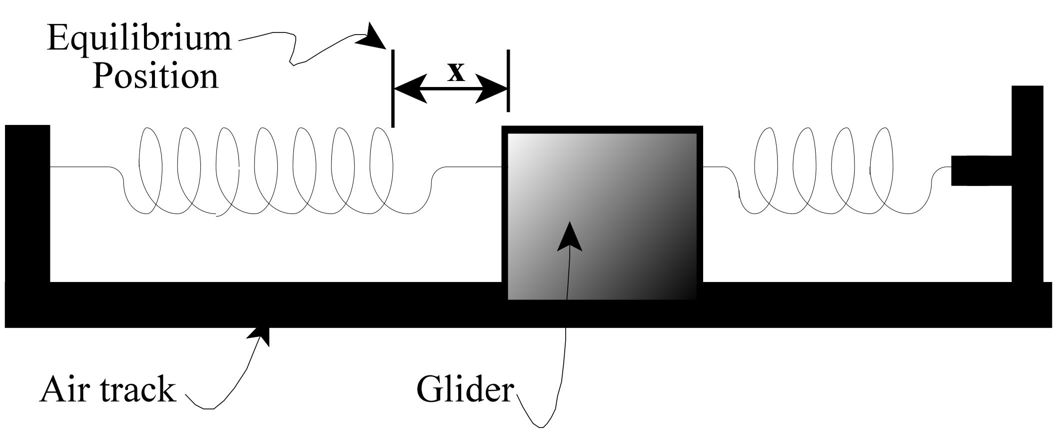
**PROCEDURE:** This experiment consists of several parts that slowly build up to an understanding of the driven, damped harmonic oscillator. As always, repeat each measurement a few times to gets some “statistics,” and *extract all the information you can from these measurements*. This experiment uses the air tracks.

An **overview** of the steps is as follows:

**I.** Compare free and damped motion on an inclined air track. The damping force is provided by magnets on the air glider. No springs. No driver.

**II.** Find the basic resonant frequency, $\omega_o$, of the air glider for a given set of springs. No damping. No driver.

**III.** Determine the effect of the damping on the resonant frequency, $\omega'$, and decay of the amplitude of the oscillations. Calculate the damping constant, $\boldsymbol{b}$. No driver.



**I. Damping of a Free Air Glider:** Magnets placed on the skirt of the air gliders are used to create the damping force. A moving magnet creates an induced current in nearby conductors, in this case the air track. This induced current creates a magnetic field that opposes the motion of the original magnet.

One simple way of observing and measuring this damping force, is to use a free glider (no springs) on an inclined air track and *measure how high the glider will rebound* with the magnets on the top of the glider and on its sides.

**Notes:**

- Remove the springs and the driving oscillator from the air track.
- Level the air tracks, then set the incline to a measure amount, ~5 mm.

**II. The Simple Harmonic Oscillator:** Before reconnecting the springs, this is a good time to measure the mass of the glider. Measure the period and thus the frequency of oscillation for the simple harmonic oscillator formed by the glide and two spring. A single photogate timer will be used for this. Also measure amplitude of the oscillations as it *decays,* i.e. gets smaller. Time the system until it stops. Calculate the spring constant, $\boldsymbol{k}$.

**Notes:**

- Level the air track.
- Magnets on top of the glider
- Reinstall the springs and the driving oscillator on the air track.
- Repeat a few times to get some "statistics."

**III. The Damped Harmonic Oscillator:** Repeat the above with magnets on skirts. *Compare the period and the decay of the amplitude* for the free and damped harmonic oscillator. *Plot* the decaying amplitudes. *What is the shape of the curve.* Possibly try plotting the decaying amplitude on semi-log paper. Explain. Calculate the damping constant, $\boldsymbol{b}$. *Is your measured value of* $\omega'$ *consistent with eq 7?*

**EQUIPMENT:**

Air track
Air glider
2 springs
Sonic Rangers + TI83 + CBL +cables
Photogate timer
Semi-log paper
Air supply for air track

**Equipment notes:**

*The magnets should be part of the glider system, either on the sides or on the top, throughout the experiment. WHY?*

To measure period the photogates should be in the PULSE mode, and MEMORY ON. The student can determine if 1 or 0.1 millisecond resolution is appropriate for this experiment.

Please do not slide the glider on the air track without the air blower being on.

**THEORY and ANALYSIS:** A review of the equations of some of the pieces of the driven, damped, harmonic oscillator. Recall the relationships between, period, **T**; frequency, $\nu$; and angular frequency, $\omega$**:**

$$\omega = 2\pi\nu = 2\pi\frac{1}{T} \qquad (1)$$

**The Simple Harmonic Oscillator:** If a mass, $\boldsymbol{m}$, is connected to a spring with a spring constant, $\boldsymbol{k}$, and $\boldsymbol{x}$ is the distance that the spring is stretched from equilibrium, then the equation describing the motion of the mass is:

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad (2)$$

Since this equation has a second derivative in it the first thing most physicists will try as a solution is a general sinusoidal function,

$$x(t) = A\sin(\omega_o t + \phi) \qquad (3)$$

where $A$ is the amplitude, $\omega_0$ is the angular frequency and $\phi$ is a phase angle. Trying this gives:

$$\omega_o = \sqrt{k/m} \quad . \qquad (4)$$

**The Damped Harmonic Oscillator:** If the damping force, $f_D$, is proportional to the velocity, **v,** with a damping constant, $b$, then

$$f_D = -b\,\mathbf{v} \qquad (5)$$

The equation of motion for this system is:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad (6)$$

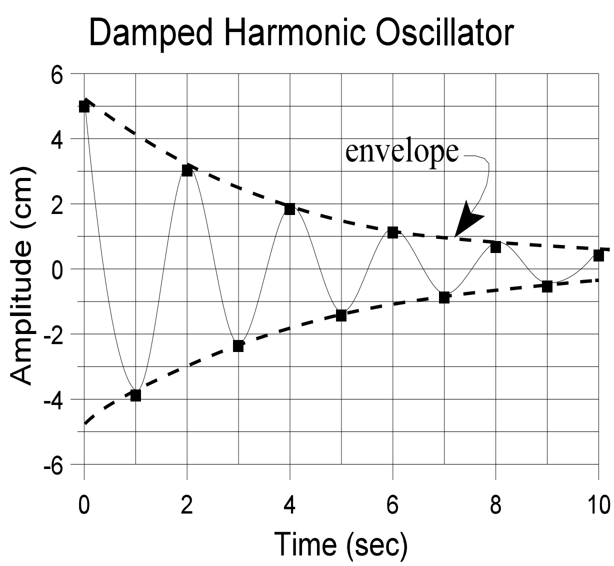
The solution to this, at least for small $b$, is

$$x = A\,e^{-bt/2m}\cos(\omega' t + \phi) \quad ,$$

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \quad , \qquad (7)$$

$$\omega' = \sqrt{\omega_o^2 - \left(\frac{b}{2m}\right)^2} \quad .$$

( As you can see this solution gets a bit odd if $b$ is large enough to make $\omega'$ imaginary.) In this solution, it is as if the amplitude of the cosine curve is confined to the envelope of a decaying exponential.




At the positive maxima in the oscillation the cosine factor in eq.7 is equal to 1 and thus one is left just the exponential factor. If one takes the natural logarithm of the remaining equation, one obtains the logarithm of the peak amplitude as a linear function of the time, with the damping constant, $b$, a factor in the slope. *Find this new equation and the equation that relates the slope to* ***b****. Use this method to determine* ***b*****.** **Note:** this method is sensitive to an accurate equilibrium point measurement from which the amplitude is measured. Why?

Since energy is always conserved, where is the initial energy of the glider when the glider stops?