Theory of the electron

The electron (cathode rays) were “scientifically discovered” by J. J. Thomson in 1896 at Cavendish Labs in Cambridge, UK. There were some very brave assertions by Thomson and his group one of which later proved to be incorrect.

These were:

1. Cathode rays are charged particles (which he called 'corpuscles').
2. These corpuscles are constituents of the atom.
3. **These corpuscles are the only constituents of the atom.**

Later on, in 1911, a brilliant experimentalist, Robert Millikan determined the charge of the electron with his famous “oil drop” experiment. After many attempts, he observed that the force due to the external field applied to each droplet was always divisible by 1.602×10^{-19}. This was the charge of one electron in Coulombs.

Remember: $F = E \times q$, in Millikan’s experiment, he knew E and F. F was $-m \times g$ (gravitational force acting on the droplet).

Then came the wave interpretation.

Niels Bohr constructed his model of the hydrogen atom assuming that electrons were waves swirling-twirling around the positively charged nucleus. This way of thinking combined with the "standing wave" concept gave rise to the prediction of discrete energy levels for hydrogen. **And it worked** !

Louis De Broglie then came up with the "wave-particle duality" interpretation for electrons (same for photons).

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v} = \frac{h}{mv}\sqrt{1 - \frac{v^2}{c^2}}$$

Classical limit

Lorentz factor for effective mass correction (fyi)

$$f = \frac{E}{h} = \frac{\gamma\, mc^2}{h} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{mc^2}{h}$$

Frequency relation to energy

Wave-particle duality

$$E = \nu h$$

Physicists often use angular frequency,

$$\omega = 2\pi\nu$$

Thus,

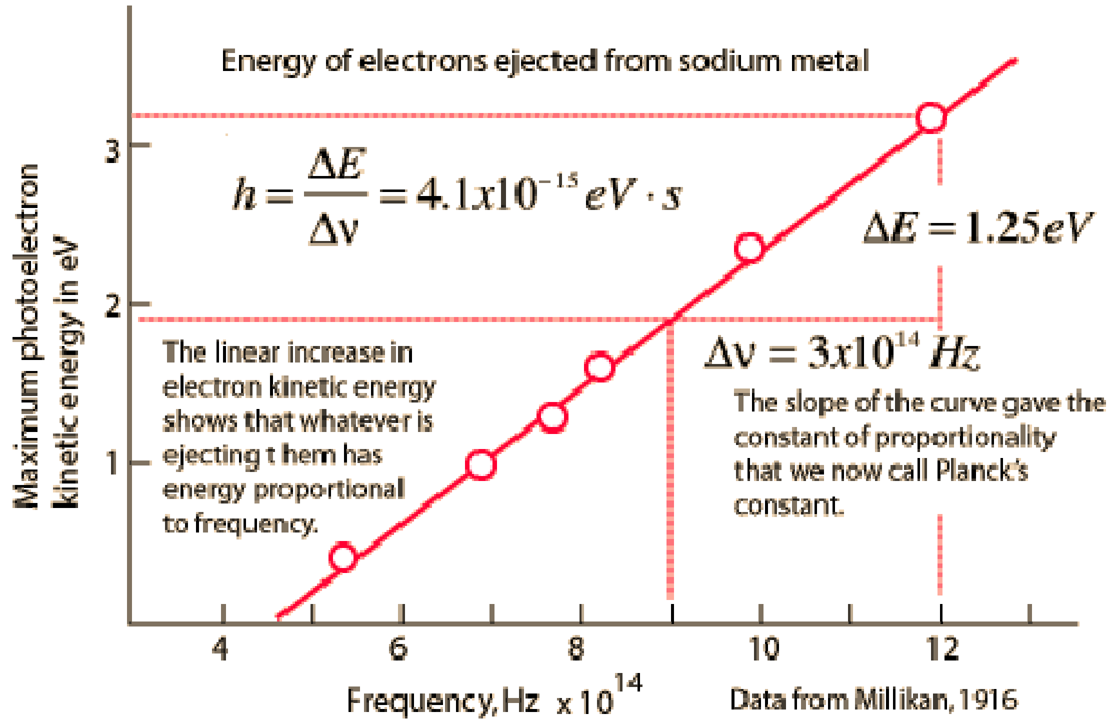
$$E = \omega\hbar$$

where

$$\hbar = \frac{h}{2\pi}$$

is called the reduced Planck constant

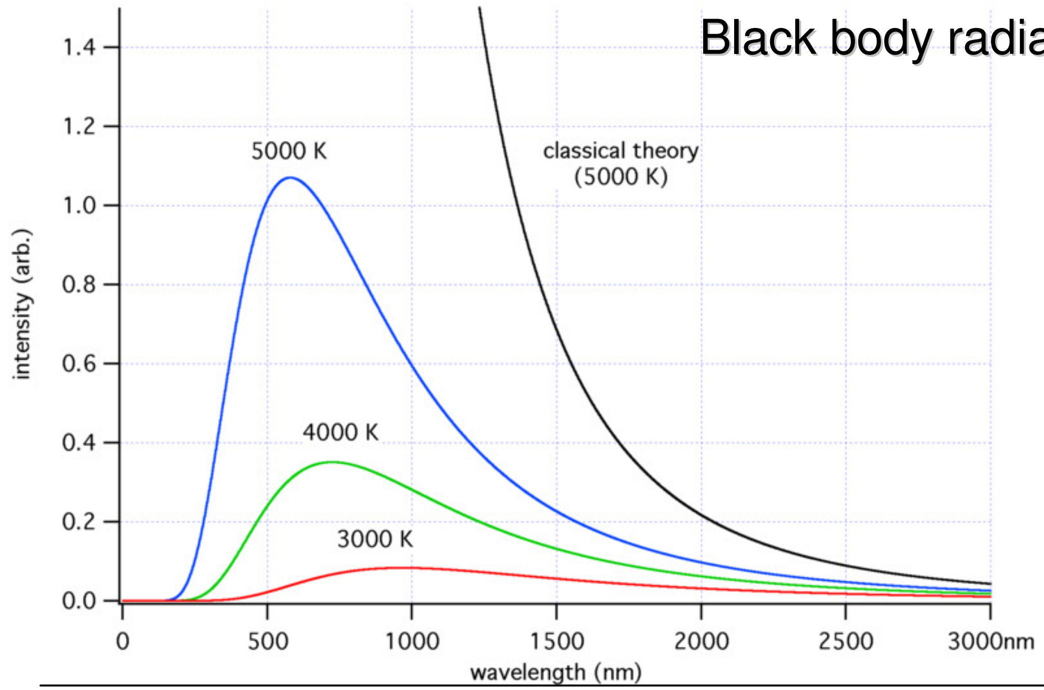
Where does the Planck Constant come from ???



Slope of electron energy-frequency curve

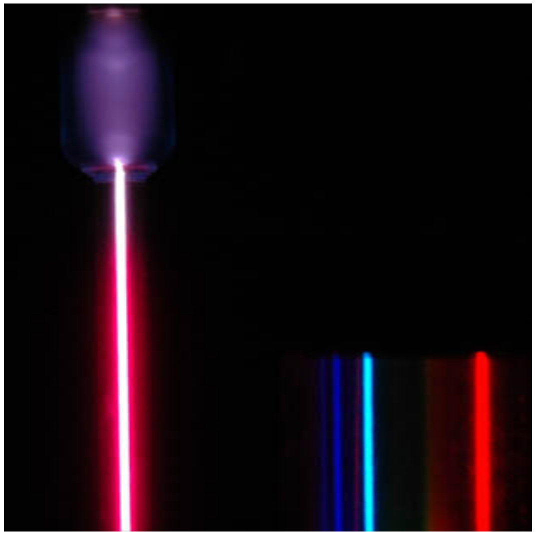
Despite a continuous variation of incoming radiation, electrons are ejected at certain energies in the "photoelectric effect" experiments.

Time to talk about the birth of quantum physics concepts



Black body radiation

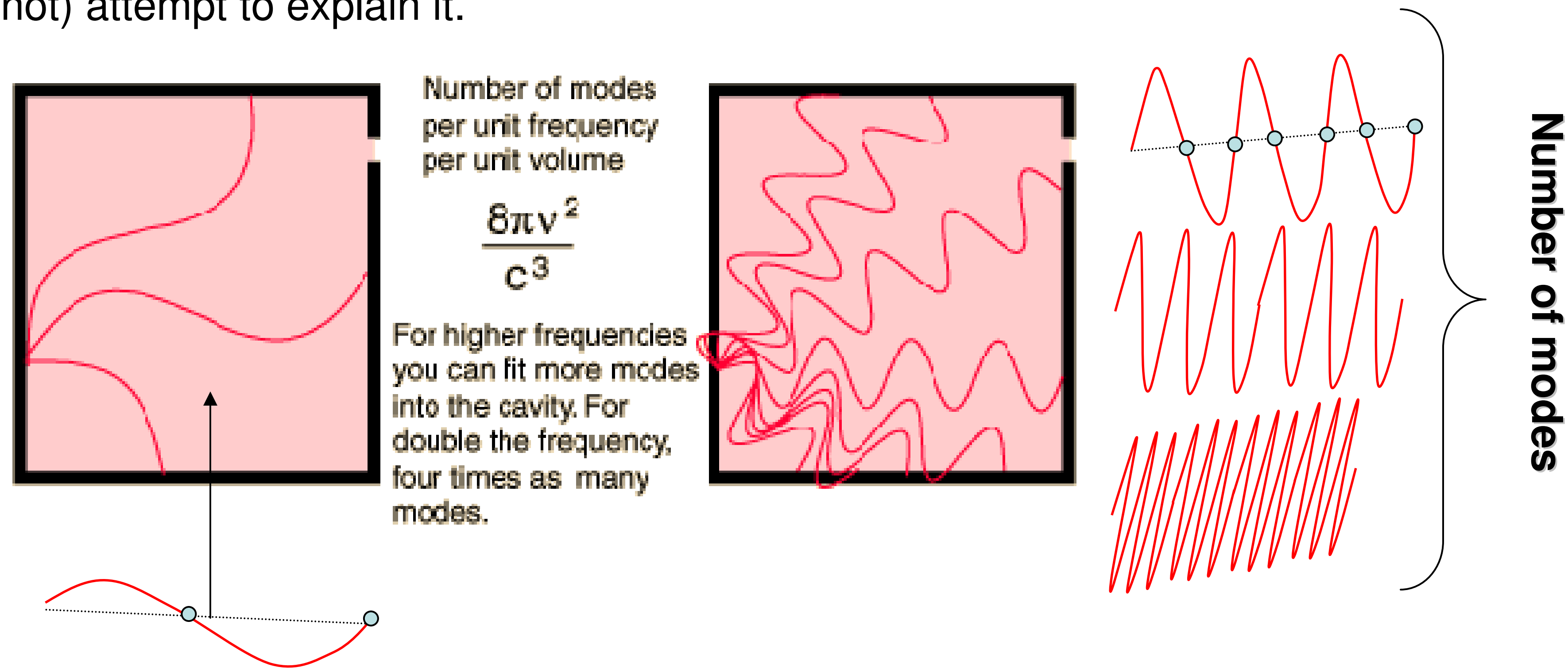
Every energy-exchange happens in discrete amounts !



Hydrogen emission spectra

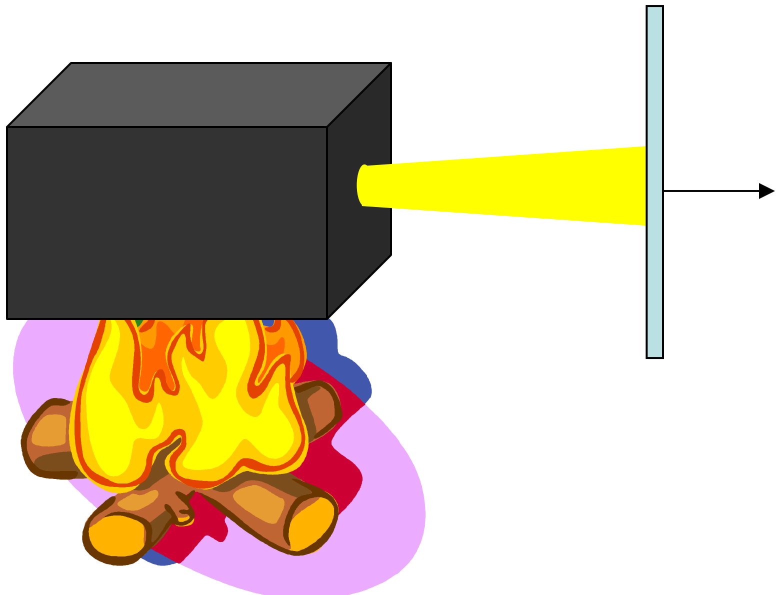
Understanding the black body radiation (Soon it will be applied to electrons!)

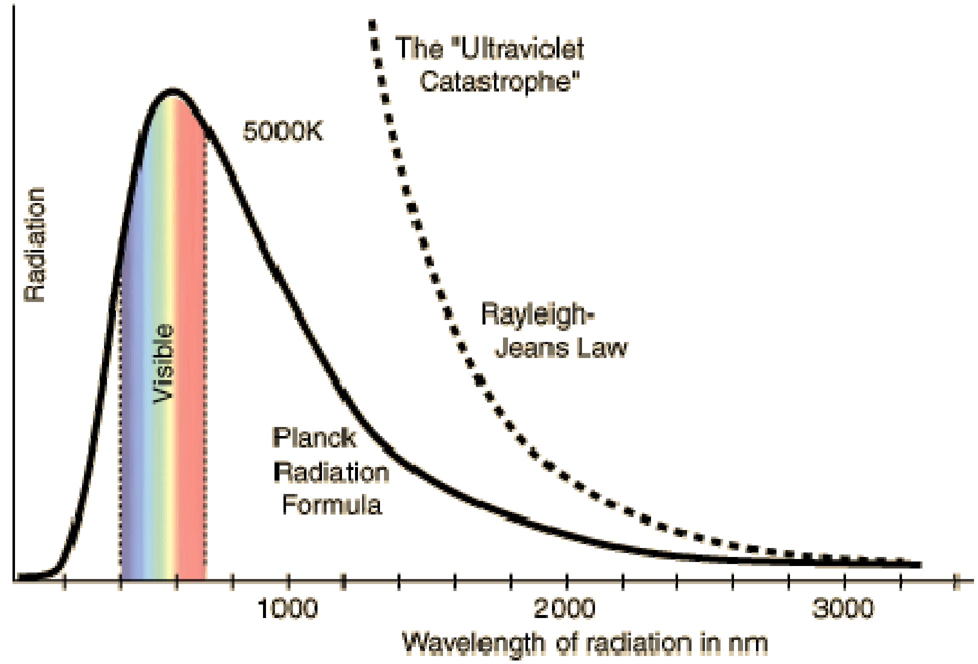
Black body radiation was the earliest puzzle to be solved. Max Planck made a ad-hoc (full of assumptions-that he did not know whether they were correct or not) attempt to explain it.

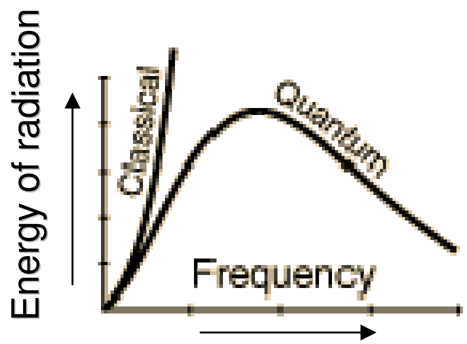


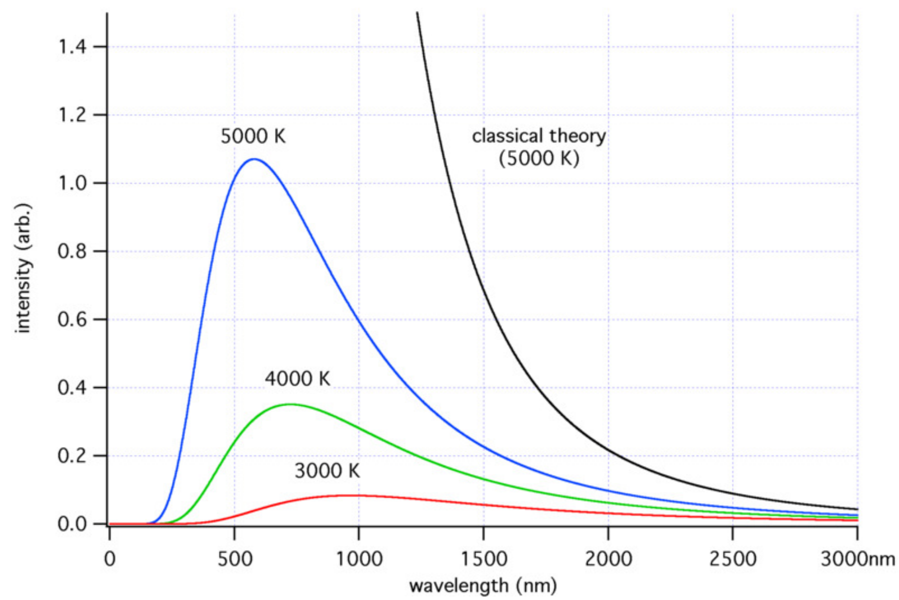
For higher frequencies, more curves can be fit with the **constraint** that the wave function will become zero at the wall (Boundary condition).

"If the mode is of shorter wavelength, there are more ways you can fit it into the cavity to meet that condition. Careful analysis by Rayleigh and Jeans showed that the number of modes was proportional to the frequency squared." (http://hyperphysics.phy-astr.gsu.edu/hbase)



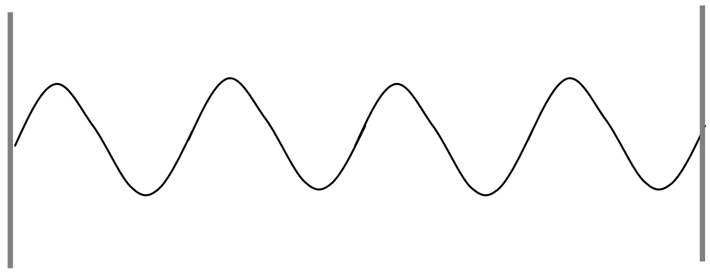

The "Ultraviolet
Catastrophe"
5000K
Radiation
Visible
Rayleigh-
Jeans Law
Planck
Radiation
Formula
1000
2000
3000
Wavelength of radiation in nm


Energy of radiation
Classical
Quantum
Frequency


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
intensity (arb.)
5000 K
classical theory
(5000 K)
4000 K
3000 K
0
500
1000
1500
2000
2500
3000nm
wavelength (nm)

What is a "standing wave"?

"…a wave that neither goes left nor right (in 1D)"



A wave whose ends are fixed

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

The wave equation in 3D

$$E = E_0 \sin\frac{n_1 \pi x}{L} \sin\frac{n_2 \pi y}{L} \sin\frac{n_3 \pi z}{L} \sin\frac{2\pi c t}{\lambda}$$

General solution

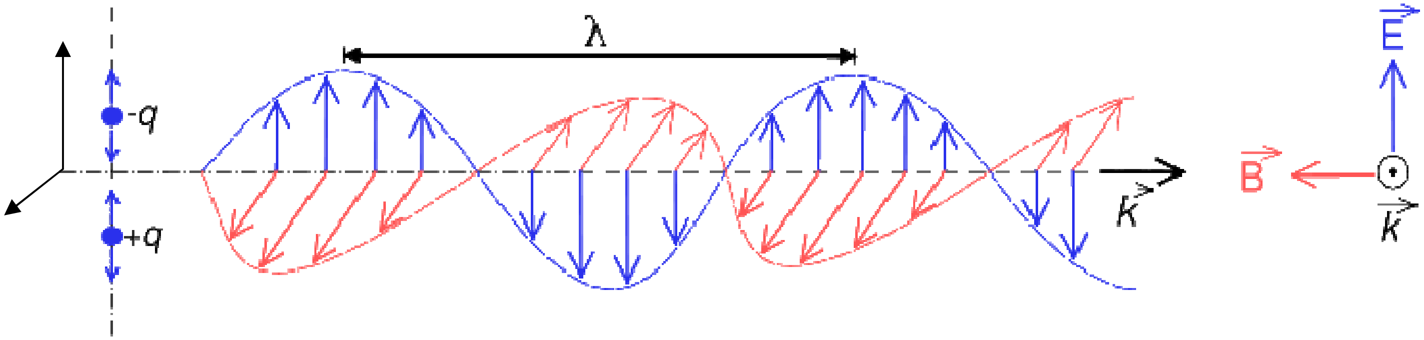
$$\left[\frac{n_1 \pi}{L}\right]^2 + \left[\frac{n_2 \pi}{L}\right]^2 + \left[\frac{n_3 \pi}{L}\right]^2 = \left[\frac{2\pi}{\lambda}\right]^2$$

When the general solution is substituted into the wave equation.

$$n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2}$$

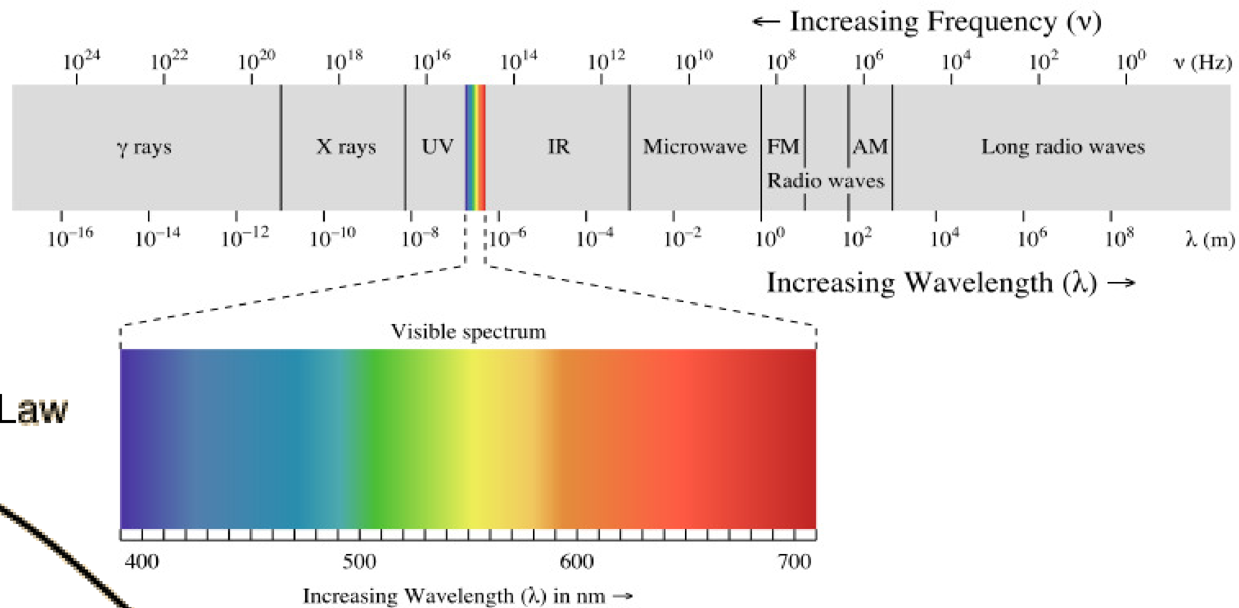
Simplify and you end up with this

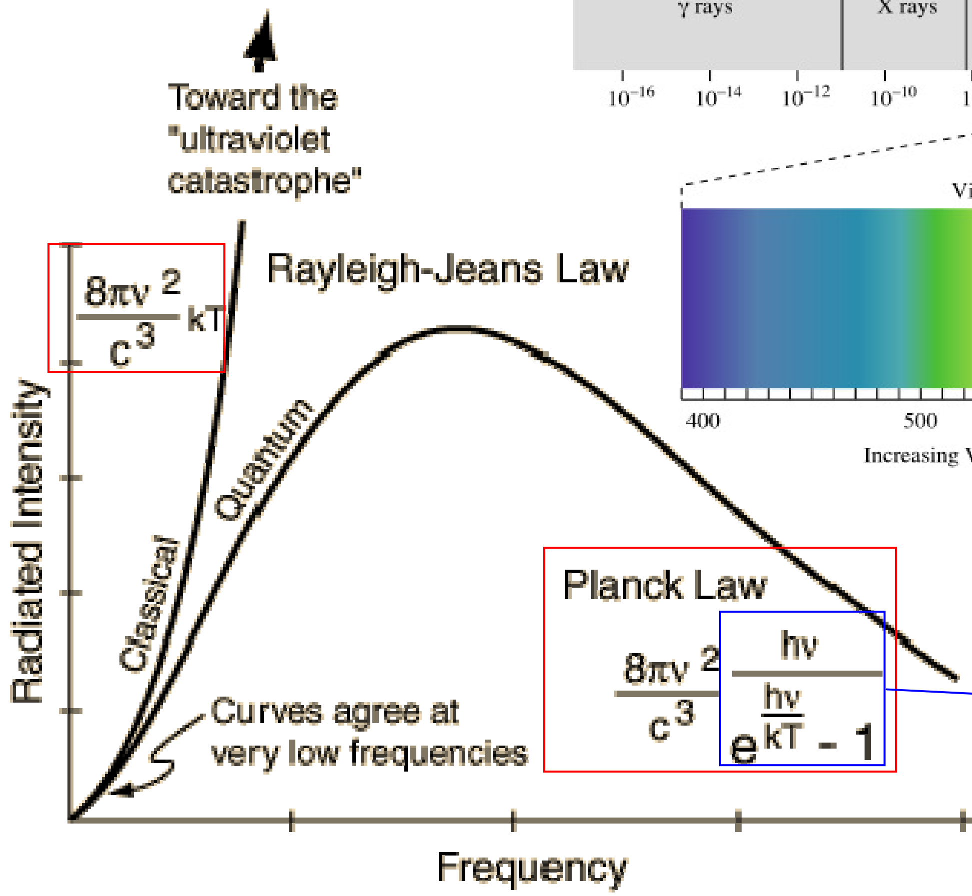
Max Planck assumed that: There are oscillators on the walls of the enclosure (inside the black body). When heated up, they oscillate. This oscillation produces electromagnetic radiation (just like the electron oscillating up and down on a antenna)



The electric field and magnetic field has to be zero at the wall (otherwise we get charging on the walls of the furnaces – who gets an electric shock when touching a hot surface?)

Remember: The shorter the wavelength, the more number of curves could be produced to fit inside the cube. These curves are each called “modes”.

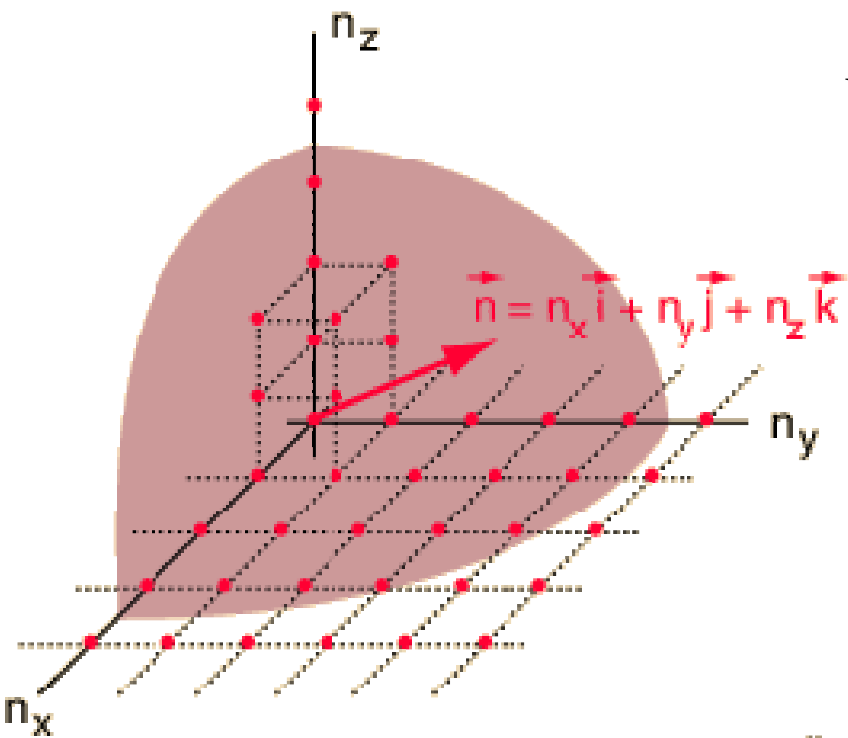




Max Planck modification
(or correction)

According to Rayleigh and Jeans, the number of modes possible to fit inside the cube goes to infinity with higher oscillator frequency.

Density of modes for a given wavelength



$$V_{Sphere} = \frac{4}{3}\pi r^3$$

$$r = (n_1^2 + n_2^2 + n_3^2)^{1/2}$$

n's are positive, so:

$$V = \frac{1}{8}\frac{4}{3}\pi r^3$$

$$\text{"volume" of } n\text{'s} = \frac{4\pi}{3}\left(n_1^2 + n_2^2 + n_3^2\right)^{3/2}$$

$$n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2}$$

This was the solution of the wave equation

$$Number\ of\ modes = N = \frac{\pi}{3}\left(n_1^2 + n_2^2 + n_3^2\right)^{3/2} = \frac{8\pi L^3}{3\lambda^3}$$

Until now, we calculated "how many standing wave modes we can fit into a cavity"

We want to know how many modes we can fit into a small infinitesmal change in the wavelength of radiation (radiation that is emitted by the oscillators on the cavity walls)

$$\frac{dN}{d\lambda} = \frac{d}{d\lambda}\left[\frac{8\pi L^3}{3\lambda^3}\right] = -\frac{8\pi L^3}{\lambda^4}$$

$$\frac{\text{Number of modes per unit wavelength}}{\text{Cavity volume}} = -\frac{1}{L^3}\frac{dN}{d\lambda} = \frac{8\pi}{\lambda^4}$$

Each mode is supposed to have an energy kT (Some thermodynamics here).

$\frac{8\pi}{\lambda^4}$ Number of modes x kT $\quad kT\frac{8\pi}{\lambda^4}$

Electronic-Magnetic-Optical Properties are all about the behavior of electrons in solids.

A few examples:

-Very weakly bound electrons with “available density of states (empty parking lot)”:

Conductors

-Unpaired electrons in terms of spins in the outer shells:

Magnetism

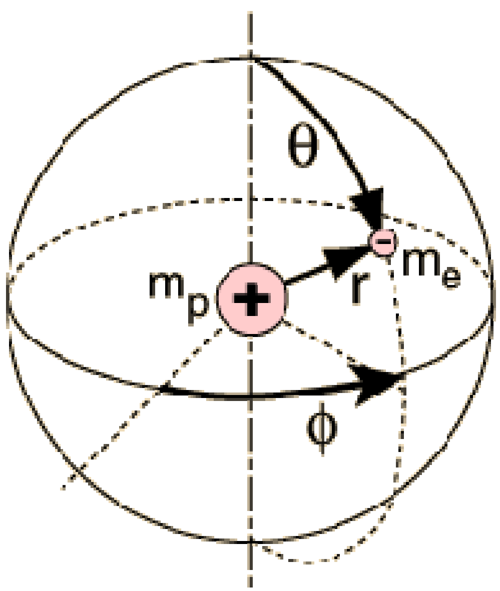
-Electrons in “bands” that cannot move anywhere (full parking lot) but some can jump to the next available/allowed band and then they have plenty of states to hop between:

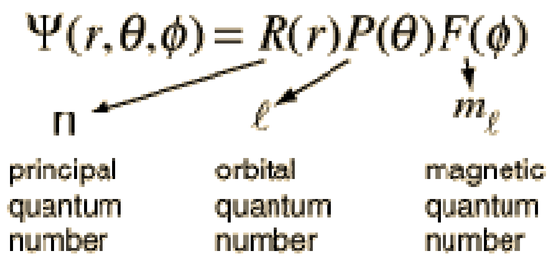
Semiconductor

-What if some electrons fall back into the previous band, the energy-state that they belonged to?:

Light emitting diode

In the solution to the **Schrodinger equation** for the hydrogen atom, three quantum numbers arise from the **space geometry of the solution** and a fourth arises from **electron spin**. No two electrons can have an **identical set of quantum numbers** according to the **Pauli exclusion principle** , so the quantum numbers set limits on the number of electrons which can occupy a given state and therefore give insight into the building up of the **periodic table of the elements**.





Some insight about the subshells (sub-energy levels)

State	Principal quantum number n	Orbital quantum number	Magnetic quantum number	Spin quantum number	Maximum number of electrons	
1s	1	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	
2s	2	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	8
2p	2	1	-1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6	
3s	3	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	18
3p	3	1	-1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6	
3d	3	2	-2,-1,0,1,2	$+\frac{1}{2}, -\frac{1}{2}$	10	

		n=1	n=2	n=3	n=4
s -- sharp	$\ell = 0$	1s	2s	3s	4s
p -- principal	$\ell = 1$		2p	3p	4p
d -- diffuse	$\ell = 2$			3d	4d
f -- fundamental	$\ell = 3$				4f

g $\ell = 4$ beyond this point, the notation just follows the alphabet

h $\ell = 5$

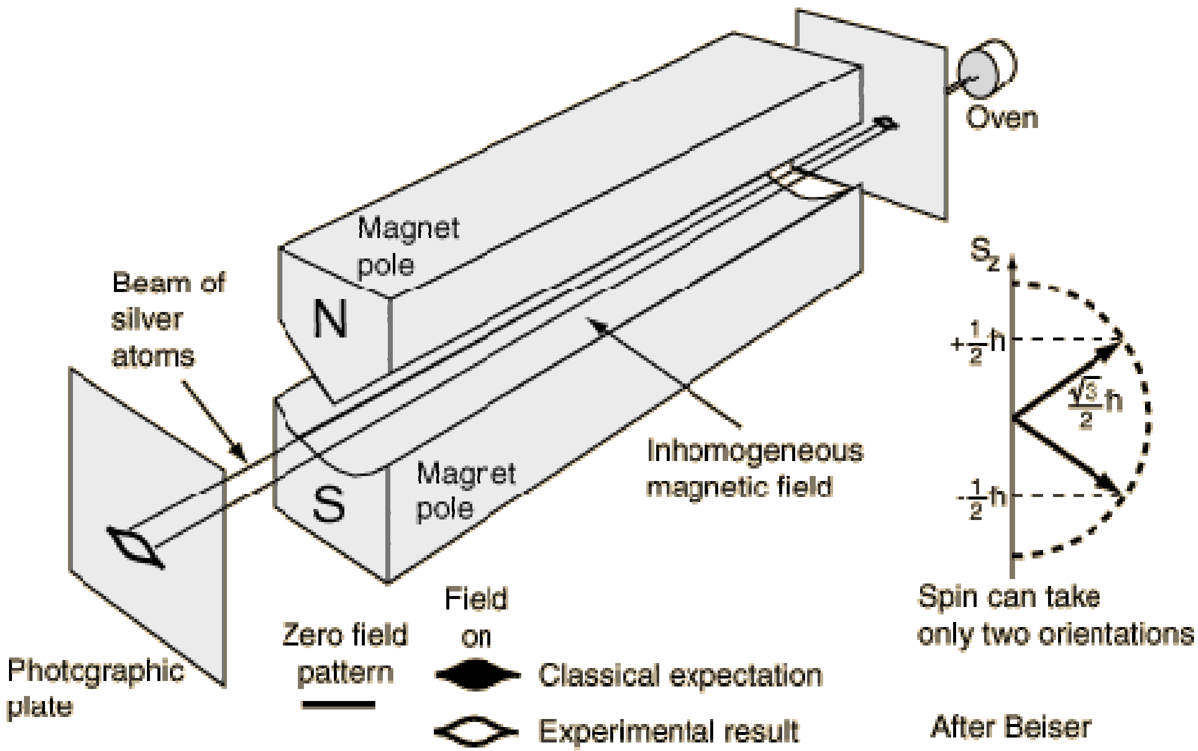
...

Electron spin

Two types of experimental evidence which arose in the 1920s suggested an additional property of the electron. One was the closely spaced splitting of the hydrogen spectral lines, called fine structure. The other was the Stern-Gerlach experiment which showed in 1922 that a beam of silver atoms directed through an inhomogeneous magnetic field would be forced into two beams. Both of these experimental situations were consistent with the possession of an intrinsic angular momentum and a magnetic moment by individual electrons. Classically this could occur if the electron were a spinning ball of charge, and this property was called electron spin.

With this evidence, we say that the electron has spin 1/2. An angular momentum and a magnetic moment could indeed arise from a spinning sphere of charge, but this classical picture cannot fit the size or quantized nature of the electron spin. The property called electron spin must be considered to be a quantum concept without detailed classical analogy.

Stern-Gerlach Experiment (to determine the electron spin)



Simple harmonic oscillator

$$mg - kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \longrightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \longrightarrow \quad \frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

$$\omega_0 = \sqrt{k/m}$$

General solution: $x(t) = x_m \cos(\omega_0 t + \phi)$

In a simple harmonic oscillator, not every oscillation frequency is allowed, just like an electron in a potential well.