pn JUNCTION
THE SHOCKLEY MODEL

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"Although the hole and its negative counterpart, the excess electron, have been prominent in the theory of solids since the work of A.H. Wilson in 1931, the announcement of the transistor in 1948 has given holes and electrons new technological significance."

William Shockley, 1950
(Nobel Laureate, 1956;
from Electrons and Holes in Semiconductors
D. van Nostrand Co. Inc., 1950)

William Shockley and his group celebrate Shockley's Nobel prize in 1956. First left, sitting, is G.E. Moore (Chairman Emeritus of Intel), standing fourth from right is R.N. Noyce, inventor of the integrated circuit, and standing at the extreme right is J.T. Last.

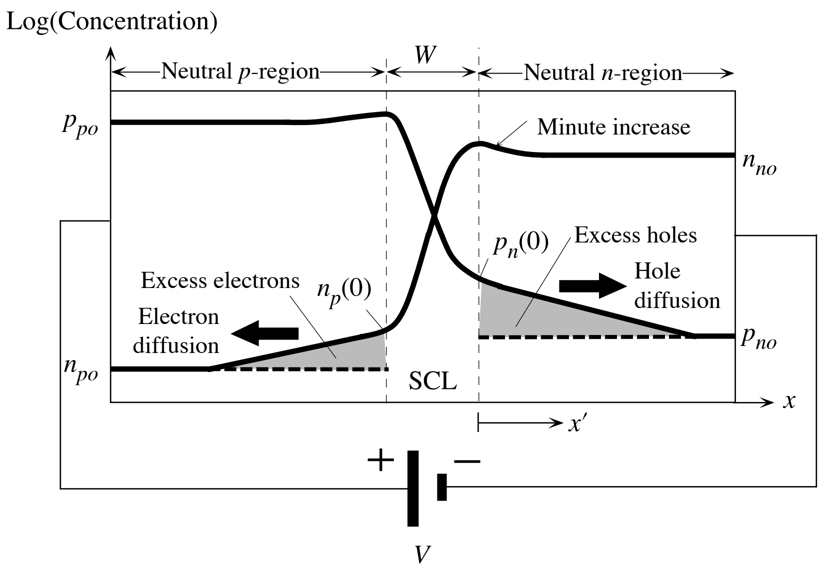
ISOURCE: W = Shockley, the Transistor Pioneer - Portrait of an Inventive Genius, P. K. Bondyopadhyay, Proceedings IEEE, Vol. 86, No 1, January 1998, p 202, Figure 16 (Courtesy of IEEE)

pn Junction *I–V* Characteristics

The current density in a forward biased *pn* junction is generally described by the Shockley equation,

$$J = \left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a}\right) n_i^2 \left[\exp\left(\frac{eV}{kT}\right) - 1\right] \qquad \textit{Shockley equation} \qquad (1)$$

where e is the electronic charge, k is Boltzmann's constant, T is temperature (K), V is the voltage across the *pn* junction, n_i is the intrinsic concentration, D is the diffusion coefficient , L is the diffusion length and N_a and N_d are the acceptor and donor doping concentrations respectively. The subscripts e and h refer to electrons and holes, respectively, as minority carriers; that is, holes in the *n*-side and electrons in the *p*-side. If τ is the charge carrier lifetime (recombination time) then $L = \sqrt{D\tau}$. The Shockley expression neglects the current component that is due to recombination in the depletion region, that is in the space charge layer (SCL). The electron and hole concentrations across the device are depicted (in an exaggerated way) in Figure 1. The application of a forward bias leads to the injection of minority carriers into the neutral regions of the diode. The minority carrier concentrations (*e.g.* holes and electron concentrations) at the space charge layer (SCL) boundaries in the neutral regions (*n*- and *p*-regions respectively) are represented as $p_n(0)$ and $n_p(0)$. In a long diode the minority carrier concentration profile falls exponentially towards the electrode, which means that there is a concentration gradient and hence diffusion. The minority carriers therefore diffuse towards the bulk giving rise to a diode current. These arguments lead to the Shockley equation stated in Equation (1) for a p^+n junction long diode.



Forward biased *pn* junction and the injection of minority carriers. Carrier concentration profiles across the device under forward bias. Note: SCL = space charge layer and *W* = width of the SCL with forward bias. Other symbols have their usual meanings.

Figure 1

The reverse current density component due to thermal generation of electron-hole pairs (EHPs) within the depletion region, as depicted in Figure 2, is given by

$$J_{gen} = \frac{eWn_i}{\tau_g} \qquad \textit{Thermal generation} \qquad (2)$$

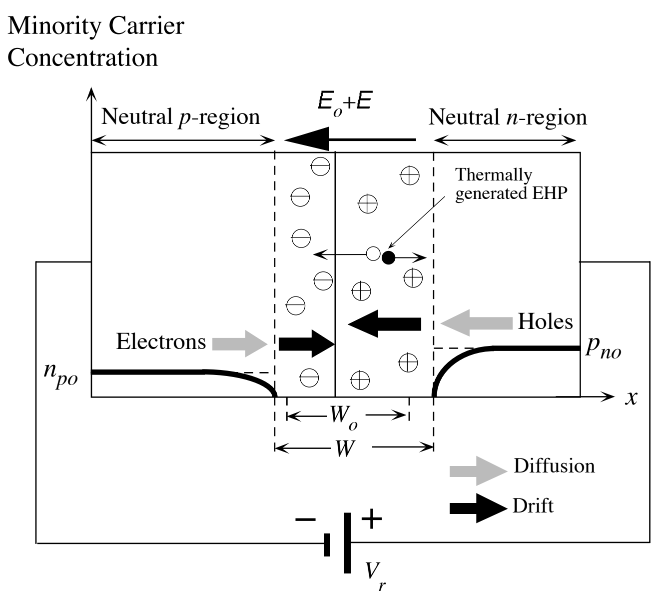
where W is the width of the depletion region and τ_g is the *mean* thermal generation time. Thermal generation of EHPs in the depletion region occurs through generation-recombination centers and depends on carrier concentrations, crystal defects and impurities. τ_g in Equation (2) represents a *mean* thermal generation time calculated by integrating the rate of thermal generation across the depletion region.

There is also a contribution to the reverse current arising from the thermal generation of minority carriers in the neutral regions within a diffusion length to the SCL, their diffusion to the SCL, and subsequent drift through the SCL (Figure 2). This is essentially the Shockley model with a negative voltage, that is Equation (1) with a reverse bias. The battery replenishes the minority carriers that are lost in this way from the neural regions. Stated differently, there is a reverse current due to the diffusion of minority carriers in neutral regions towards the SCL.

The width of the depletion region with a reverse bias $V = -V_r$ is given by

$$W = \left[\frac{2\varepsilon(N_a + N_d)(V_o + V_r)}{eN_aN_d}\right]^{1/2} \qquad \textit{Depletion layer (SCL) width} \qquad (3)$$

where $V_o = (kT/e)\ln[(N_aN_d)/n_i^2]$ is the built-in voltage, $\varepsilon = \varepsilon_o\varepsilon_r$ is the permittivity of the semiconductor material. Equation (3) assumes an abrupt *pn* junction.



Reverse biased *pn* junction. Minority carrier concentration profiles and the origin of the reverse current. Note: EHP = electron-hole pair, SCL = space charge layer, E = electric field, W = width of the SCL with reverse bias, V_r = reverse bias. Other symbols have their usual meanings. Subscript *o* refers to "no-bias" condition.

Figure 2

Problem: The *pn* junction

Consider a long Si diode made of an abrupt p^+n junction which has 10^{15} donors cm^{-3} on the *n*-side and 10^{18} acceptors on the *p*-side. The dependence of the hole and electron drift mobility on the dopant concentration is shown in Figure 3. The minority carrier recombination times are τ_h = 490 ns for holes in the *n*-side and τ_e = 23.8 ns for electrons in the p^+-side. The cross sectional area is 0.1 mm^2. Assume a long diode. The thermal generation time τ_g in the depletion region is ~1 ms. Assume that the reverse current is dominated by the thermal generation rate in the depletion region.

a Calculate the forward current at 27 °C when the voltage across the diode is 0.6 V.

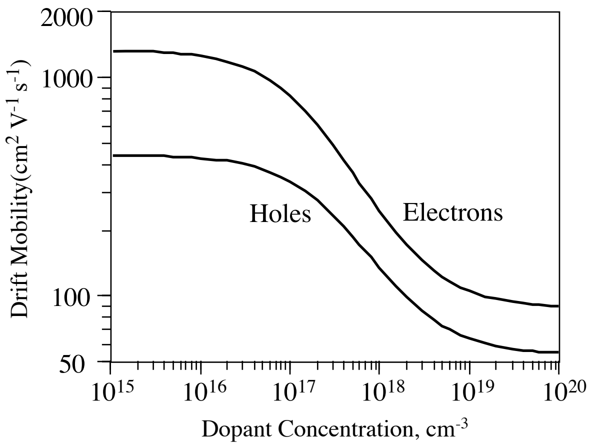
b Estimate the forward current at 57 °C when the voltage across the diode is still 0.6 V.

c Calculate the voltage across the diode at 57 °C if the forward current in **a** above at 27 °C is kept constant.

d What is the reverse current at 27 °C when the diode voltage is −5 V?

e Estimate the reverse current at 57 °C when the diode voltage is −5 V.

Note: Assume that the forward current is determined by the Shockley equation (minority carrier diffusion).



The variation of the drift mobility with dopant concentration in Si for electrons and holes.

Figure 3

Solution

a Consider room temperature, $T = T_1 = 300$ K. $kT/e = kT_1/e = 0.2585$.

The general expression for the diffusion length is $L = \sqrt{D\tau}$ where D is the diffusion coefficient and τ is the carrier lifetime. D is related to the mobility of carriers, μ, via the Einstein relationship, $D/\mu = kT/e$. We therefore need to know μ to calculate D and hence L. Electrons diffuse in the p-region and holes in the n-region so that we need μ_e in the presence of N_a acceptors and μ_h in the presence of N_d donors. From the drift mobility, μ vs. dopant concentration for silicon graph we have the following:

$$N_a = 10^{18}\text{ cm}^{-3},\ \mu_e \approx 250\text{ cm}^2\text{ V}^{-1}\text{ s}^{-1}$$

$$N_d = 10^{15}\text{ cm}^{-3},\ \mu_h \approx 450\text{ cm}^2\text{ V}^{-1}\text{ s}^{-1}$$

Thus, $$D_e = kT\mu_e/e \approx (0.02585\text{ V})(250\text{ cm}^2\text{ V}^{-1}\text{ s}^{-1}) = 6.46\text{ cm}^2\text{ s}^{-1}$$

and $$D_h = kT\mu_h/e \approx (0.02585\text{ V})(450\text{ cm}^2\text{ V}^{-1}\text{ s}^{-1}) = 11.63\text{ cm}^2\text{ s}^{-1}$$

Diffusion lengths are

$$L_e = \sqrt{D\tau_e} = \sqrt{\left(6.46\text{ cm}^2\text{ s}^{-1}\right)\left(23.8\times10^{-9}\text{ s}\right)} = 3.93 \times 10^{-4}\text{ cm, or } 3.93\ \mu\text{m}$$

and $$L_h = \sqrt{D\tau_h} = \sqrt{\left(11.63\text{ cm}^2\text{ s}^{-1}\right)\left(490\times10^{-9}\text{ s}\right)} = 2.39 \times 10^{-3}\text{ cm, or } 23.9\ \mu\text{m}$$

The built-in potential is

$$V_o = (kT/e)\ln(N_dN_a/n_i^2) = (0.02585\text{ V})\ln[(10^{18}\text{ cm}^{-3} \times 10^{15}\text{ cm}^{-3})/(1.45 \times 10^{10}\text{ cm}^{-3})^2]$$

$$V_o = 0.755\text{ V}$$

To calculate the forward current when V = 0.6 V, we need to evaluate both the diffusion and recombination components of the current. It is likely that the diffusion component will exceed the recombination component at this forward bias (this can be easily verified). Assuming that the forward current is due to minority carrier diffusion in neutral regions,

$$I = I_{so}[\exp(eV/kT) - 1] \approx I_{so}\exp(eV/kT) \text{ for } V >> kT/e \ (\approx 26 \text{ mV})$$

where,

$$I_{so} = AJ_{so} = Aen_i^2[(D_h/L_hN_d) + (D_e/L_eN_a)] \approx Aen_i^2D_h/L_hN_d$$

as $N_a >> N_d$. In other words, the current is mainly due to the diffusion of holes in the n-region. Thus,

$$I_{so} = \frac{(0.1\times10^{-2}\ \text{cm}^2)(1.6\times10^{-19}\ \text{C})(1.45\times10^{10}\ \text{cm}^{-3})^2(11.66\ \text{cm}^2\ \text{s}^{-1})}{(23.9\times10^{-3}\ \text{cm})(1\times10^{15}\ \text{cm}^{-3})}$$

$$I_{so} = 1.64 \times 10^{-13} \text{ A or } 0.164 \text{ pA}$$

The forward current is then

$$I = I_{so}\exp(eV/kT_1) = (1.64 \times 10^{-13} \text{ A})\exp[(0.6 \text{ V})/(0.02585 \text{ V})]$$

$$I = 0.00197 \text{ A or } 1.97 \text{ mA}$$

b Consider $T = T_2$ = 57 K + 273 K = 330 K, kT_2/e = 0.02844. First, find the new n_i from $n_i = (N_cN_v)^{1/2}\exp(-E_g/2kT)$. Thus,

$$\frac{n_i(T_2)}{n_i(T_1)} = \frac{T_2^{3/2}\exp\left(-\frac{E_g}{2kT_2}\right)}{T_1^{3/2}\exp\left(-\frac{E_g}{2kT_1}\right)}$$

substituting, T_1 = 300 K, T_2 = 330 K, n_i(300 K) = 1.45 × 10^{10} cm^{-3}, E_g = 1.1 eV, we find n_i (330 K) = 1.16 × 10^{11} cm^{-3}. Assuming that the temperature dependence of n_i dominates those of other terms in I_{so}, the new I'_{so} is,

$$I'_{so} = I_{so}\left[\frac{n_i(T_2)}{n_i(T_1)}\right]^2$$

i.e.

$$I'_{so} = (1.64\times10^{-13}\ \text{A})\left[\frac{1.16\times10^{11}\ \text{cm}^{-3}}{1.45\times10^{10}\ \text{cm}^{-3}}\right]^2 = 1.05 \times 10^{-11} \text{ A}$$

The forward current is then

$$I' = I'_{so}\exp(eV/kT_2) = (1.05 \times 10^{-11} \text{ A})\exp[(0.6 \text{ V})/(0.02844)] = 0.0152 \text{ A}$$

c Suppose that the current is kept constant through the pn junction from 27 °C to 57 °C, that is, $I' = I$ = 0.00197 A. Suppose that the voltage across the pn junction is now V'. Then,

$$I'_{so}\exp(eV'/kT_2) = I = 0.00197 \text{ A}$$

Thus,

$$V' = (kT_2/e)\ln(I/I'_{so}) = (0.02844 \text{ V})\ln[(0.00197 \text{ A})/(1.05 \times 10^{-11})] = 0.542 \text{ V}$$

Notice that the voltage across the pn junction decreases with temperature when the current through it is kept constant.

d When a reverse bias of V_r is applied, the potential difference across the depletion region becomes $V_o + V_r$ and the width W of the depletion region is

$$W = \left[\frac{2\varepsilon(N_a + N_d)(V_o + V_r)}{eN_aN_d}\right]^{1/2} \approx \left[\frac{2\varepsilon(V_o + V_r)}{eN_d}\right]^{1/2}$$

$$W = \left[\frac{2(11.9)(8.85\times10^{-12}\ \text{F m}^{-1})(0.755\ \text{V} + 5\ \text{V})}{(1.6\times10^{-19}\ \text{C})(10^{15}\ \text{cm}^{-3}\times10^{6}\ \text{m}^{-3}/\text{cm}^{-3})}\right]^{1/2}$$

i.e. $W \approx 2.75 \times 10^{-6}$ m or 2.75 μm

The thermal generation current I_{gen} with $V_r = 5$ V is,

$$I_{gen} = \frac{eAWn_i}{\tau_g}$$

$$I_{gen} = \frac{(1.6\times10^{-19}\ \text{C})(0.001\ \text{cm}^2)(2.75\times10^{-4}\ \text{cm})(1.45\times10^{10}\ \text{cm}^{-3})}{(1\times10^{-3}\ \text{s})}$$

$I_{gen} = 6.39 \times 10^{-13}$ A or 0.639 pA

The total reverse current is due to thermal generation in the depletion region and diffusion in the neutral regions.

$I_{rev} = I_{gen} + I_{so} = 0.639$ pA + 0.164 pA = 0.80 pA

e Estimation of the reverse current at 57 °C is difficult because we need to know the temperature dependence of τ_g. If τ_g were to remain very roughly the same then $I_{gen} \propto n_i$ and the new thermal generation current would be,

$$I'_{gen} = I_{gen}\frac{n_i(330\ \text{K})}{n_i(300\ \text{K})} = (0.639\ \text{pA})\frac{(1.16\times10^{11}\ \text{cm}^{-3})}{(1.45\times10^{10}\ \text{cm}^{-3})} = 5.10\ \text{pA}$$

However, the reverse saturation current $I_{so} \propto n_i^2$ which leads to I'_{so} = 10.5 pA at 57 °C as calculated above. It is clear that the diffusion component is now greater than the thermal generation component. The total reverse current at 57 °C.

$I'_{rev} = I'_{gen} + I'_{so}$ = 5.1 pA +10.5 pA = 15.6 pA

NOTATION

A	cross sectional area of device
D_h	hole diffusion coefficient (m^2 s^{-1}) in the *n*-side
e	electronic charge (1.6×10^{-19} C)
e	(subscript) electron
EHP	electron hole pair
I	diode current
I'	new diode current (different temperature)
J	total current density
I_{so}	reverse saturation current in the Shockley model (minority carrier diffusion)

I_{gen}	reverse current due to thermal generation in the SCL
I_{rev}	total reverse current
k	Boltzmann constant ($k = 1.3807 \times 10^{-23}$ J K^{-1})
kT/e	0.0259 V at ~300 K
L_h	hole diffusion length (m) in the n-side
N_d, N_a	donor and acceptor concentrations (m^{-3})
n_i	intrinsic concentration
n_{no}, p_{po}	equilibrium majority carrier concentrations: $n_{no} = N_d$ and $p_{po} = N_a$
n_{po}, p_{no}	equilibrium minority carrier concentrations
SCL	space charge layer or depletion layer; region around the metallurgical junction that has been depleted of its normal concentrations of carriers
T	absolute temperature (K)
V	applied voltage
V_o	built-in voltage
V_r	reverse bias voltage, $V = -V_r$
W	width of depletion layer with applied voltage
ε	permittivity of a medium; $\varepsilon = \varepsilon_o\varepsilon_r$, where ε_o and ε_r are the absolute and relative permittivities
μ_h	drift mobility of holes in the n-side
τ_h	hole recombination lifetime (s) in the n-side
τ_g	mean thermal generation time in the SCL

USEFUL DEFINITIONS

Diffusion is the flow of particles of a given species from high to low concentration regions by virtue of their random motions. Diffusion flux Γ (number of particles diffusing through unit area per unit time) obeys Fick's first law, $\Gamma = -D(dn/dx)$ where D is the diffusion coefficient and dn/dx is the concentration gradient.

Excess concentration is the excess concentration above the thermal equilibrium value. Consider holes in an n-type semiconductor. If Δp_n = excess hole concentration, p_n is the instantaneous hole concentration and p_{no} is the equilibrium carrier concentration, then $\Delta p_n = p_n - p_{no}$.

Law of the junction relates the injected minority carrier concentration just outside the depletion layer to the applied voltage. For holes in the n-side, it is $p_n(0) = p_{no}\exp(eV/kT)$, where $p_n(0)$ is the hole concentration just outside the depletion layer in the n-side.

Majority carriers are electrons in an n-type and holes in a p-type semiconductor.

Mass action law in semiconductor science refers to the law $np = n_i^2$ which is valid under thermal equilibrium conditions and in the absence of external bias voltages and illumination. If n_{no} is the equilibrium majority carrier concentration and p_{no} is the equilibrium minority carrier concentration (both in an n-type semiconductor) then $n_{no}p_{no} = n_i^2$ and therefore $p_{no} = n_i^2/N_d$.

Minority carrier diffusion length (L) is the mean distance a minority carrier diffuses before recombination, $L = \sqrt{(D\tau)}$ where D is the diffusion coefficient and τ is the minority carrier lifetime.

Minority carriers are electrons in a p-type and holes in an n-type semiconductor.

Recombination of an electron hole pair involves an electron in the conduction band (CB) falling in energy down into an empty state (hole) in the valence band (VB) to occupy it. The result is the annihilation of the EHP. The recombination process may be direct or indirect, depending on the semiconductor. In direct recombination (as in GaAs), the electron falls directly from the CB into a hole in the VB. In indirect recombination (as in Si), one of the carriers, for example the electron in the CB, is first captured by a recombination center such as a crystal defect or an impurity. The other carrier (a hole in the VB) then arrives at the recombination center and recombines with the captured carrier (electron). Thus, the electron first falls into a localized energy level (at the recombination site) in the bandgap. When a hole in the VB is in the neighborhood of the recombination center, the electron falls into this hole, resulting in an indirect recombination process.

Recombination current flows under forward bias to replenish the carriers recombining in the space charge (depletion) layer. Typically it is described by $I = I_{ro}[\exp(eV/2kT) - 1]$.

Shockley diode equation relates the diode current to the diode voltage through $I = I_o[\exp(eV/kT) - 1]$. It is based on the injection and diffusion of injected minority carriers by the application of a forward bias.

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