

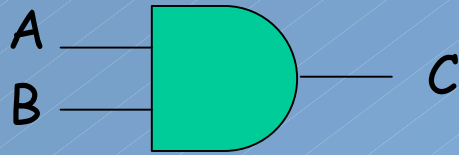
# *VHDL*

## *Logic Design Overview*

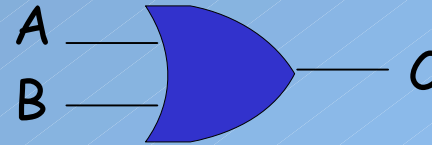
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# Combinational Logic

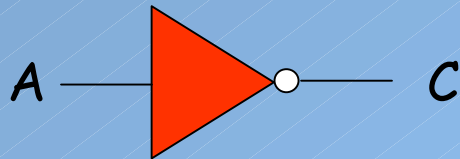
- Basic Gates



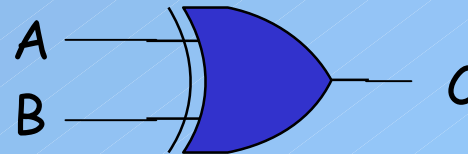
AND:  $C = AB$



OR:  $C = A+B$



NOT:  $C = A'$



XOR:  $C = A \oplus B$

# Truth Tables and Algebraic Expressions

- Full Adder



X	Y	Cin	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$\text{Sum} = X' Y' \text{Cin} + X' Y \text{Cin}' + X Y' \text{Cin}' + X Y \text{Cin}$$

$$\text{Cout} = X' Y \text{Cin} + X Y' \text{Cin} + X Y \text{Cin}' + X Y \text{Cin}$$

# *Minterm and Maxterm Expansions*

- Minterm expansion

$$\text{Sum} = X' Y' C_{in} + X' Y C_{in}' + X Y' C_{in}' + X Y C_{in} = \sum m(1, 2, 4, 7)$$

$$\text{Cout} = X' Y C_{in} + X Y' C_{in} + X Y C_{in}' + X Y C_{in} = \sum m(3, 5, 6, 7)$$

- Maxterm expansion

$$\text{Sum} = (X+Y+C_{in})(X+Y'+C_{in}')(X'+Y+C_{in}')(X'+Y'+C_{in}) = \prod M(0, 3, 5, 6)$$

$$\text{Cout} = (X+Y+C_{in})(X+Y+C_{in}')(X+Y'+C_{in})(X'+Y+C_{in}) = \prod M(0, 1, 2, 4)$$

- Minterm expansion  $\Leftrightarrow$  Maxterm expansion

# Algebraic Simplification 1

## 1. Combining Terms ( $XY + XY' = X$ )

$$-ABC'D' + ABCD' = ABD'(C + C') = ABD'$$

$$\begin{aligned} -C_{out} &= X'Y C_{in} + XY' C_{in} + XY C_{in}' + XY C_{in} \\ &= X'Y C_{in} + XY' C_{in} + XY C_{in}' + XY C_{in} + \textcolor{red}{XY C_{in}} + \textcolor{red}{XY C_{in}} \\ &= X'Y C_{in} + \textcolor{red}{XY C_{in}} + XY' C_{in} + \textcolor{red}{XY C_{in}} + XY C_{in}' + XY C_{in} \\ &= Y C_{in} + X C_{in} + XY \end{aligned}$$

## 2. Eliminating Terms

$$-A'B + A'BC = A'B$$

$$-\text{Consensus theorem: } XY + X'Z + YZ = XY + X'Z$$

$$CBD + C'A'B + DA'B = CBD + C'A'B + BDA'B = CBD + CA'B'$$

## *Algebraic Simplification 2*

- Eliminating Literals

- $X + X'Y = X + Y$

- $A'B + A'B'C'D' + ABCD' = A'(B + B'C'D') + ABCD'$   
 $= A'(B + C'D') + ABCD' = A'B + A'C'D' + ABCD'$   
 $= B(A' + ACD') + A'C'D'$   
 $= B(A' + CD') + A'C'D' = A'B + BCD' + A'C'D'$

- Adding Redundant Terms

- Adding  $XX'$ , multiplying by  $(X+X')$ ,

- Adding  $YZ$  to  $XY + X'Z$  or adding  $XY$  to  $X$

- $AB + AD + A'C' + DB'C' = AB + DC'B' + AD + A'C' + DC'$   
 $= AB + AD + A'C' + DC' = AB + AD + A'C'$

# Duality Principle

- Important property of Boolean algebra
  - If  $x + 0 = x$  then  $x \cdot 1 = x$

$x + x = x$	$x \cdot x = x$
$x + x' = 1$	$x \cdot x' = 0$
$x + 1 = 1$	$x \cdot 0 = 0$
$(x + y + \dots)' = x' \cdot y' \dots$	$(x \cdot y \cdot \dots)' = x' + y' + \dots$
$x + x \cdot y = x$	$x \cdot (x + y) = x$
$xy + x'z + yz = xy + x'z$	$(x+y)(x'+z)(y+z) = (x+y)(x'+z)$
$(x+y)(x'+z) = xz + x'y$	$xy + x'z = (x+z)(x'+y)$

# Exclusive OR (XOR)

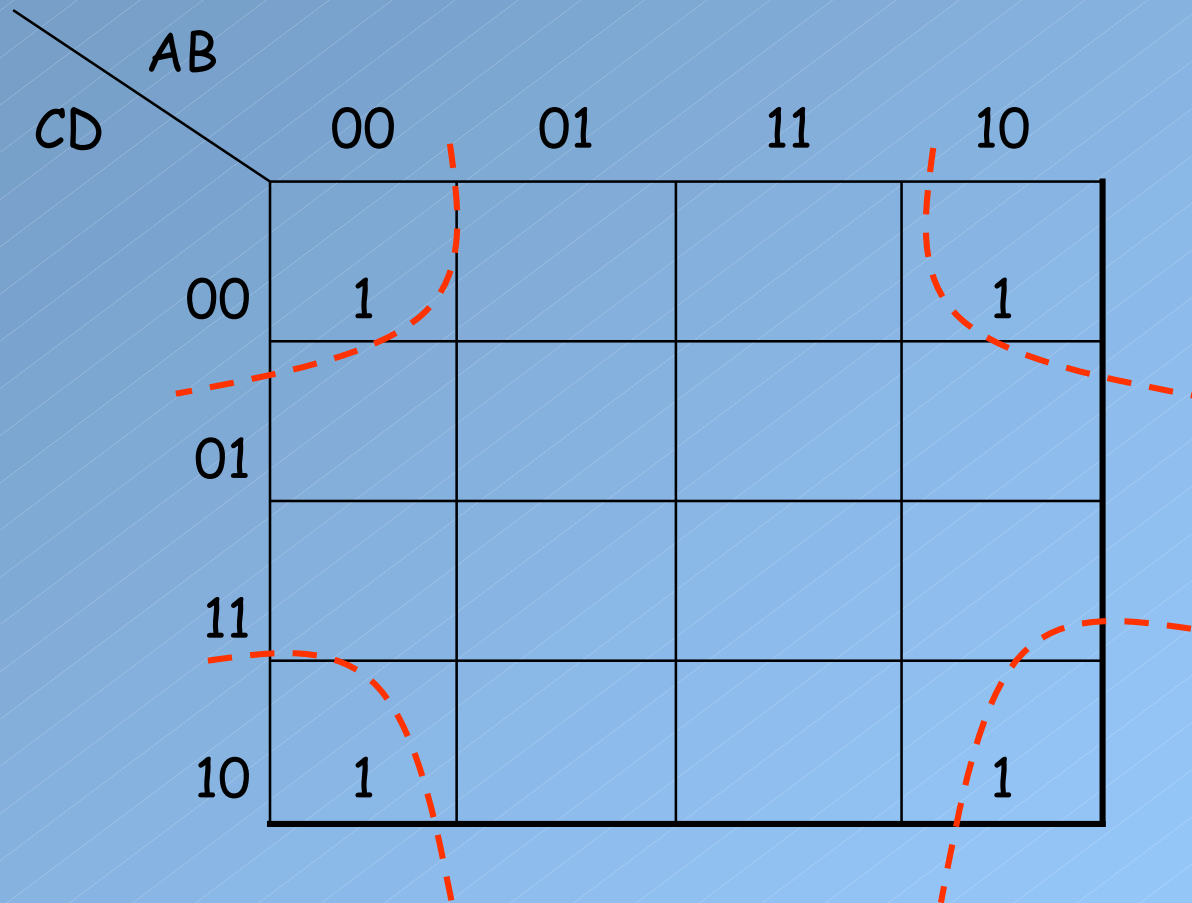
- Some rules

- $X \oplus 0 = X$
- $X \oplus 1 = X'$
- $X \oplus X = 0$
- $X \oplus X' = 1$
- Commutative, associative, and distributive laws hold.
- $X \oplus Y = X'Y + XY'$
- $(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$
- Sum =  $X'Y' Cin + X'Y Cin' + XY' Cin' + XY Cin$   
=  $(X'Y' + XY) Cin + (X'Y + XY') Cin'$   
=  $(X \oplus Y)' Cin + (X \oplus Y) Cin'$   
=  $X \oplus Y \oplus Cin$



# Karnaugh Maps (1)

- $F = A'B'C'D' + AB'C'D' + A'B'CD' + AB'CD'$



- $F = A'B'C'D' + AB'C'D' + A'B'CD' + AB'CD' = B'D'$

## Karnaugh Maps (2)

- $F = A'B'C'D' + AB'C'D' + A'B'CD' + AB'CD'$

		AB			
		00	01	11	10
CD	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

- $F = (B+D)' = B'D' \text{ (DeMorgan's Law)}$

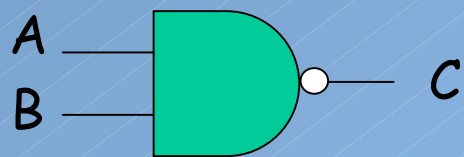
## Karnaugh Maps (3)

AB \ CD		AB			
		00	01	11	10
CD	00	1	X	0	1
	01	1	1	0	0
	11	0	1	1	1
	10	1	0	X	0

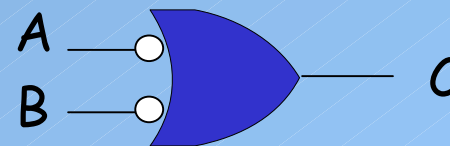
- $F = A'C' + A'B'D' + ACD + BCD$  or
- $F = A'C' + A'B'D' + ACD + A'BD$

# Designing with NAND and NOR gates - 1

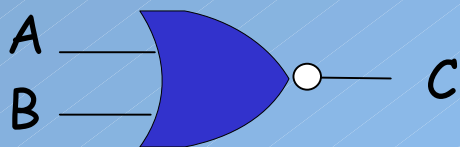
- In many technologies, implementation of NAND or NOR gates is easier than that of AND or OR gates
  - Any logic function can be realized using only NAND gates or only NOR gates



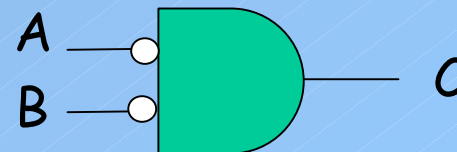
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NAND:  $C = (AB)' = A' + B'$



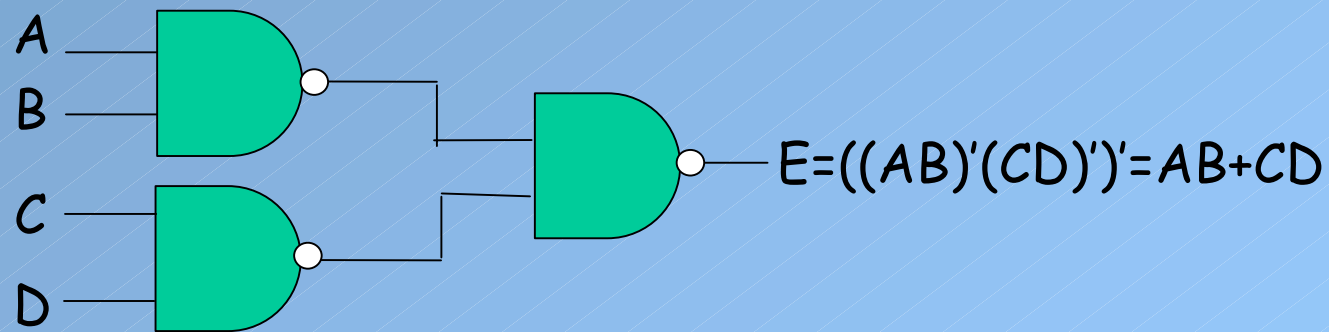
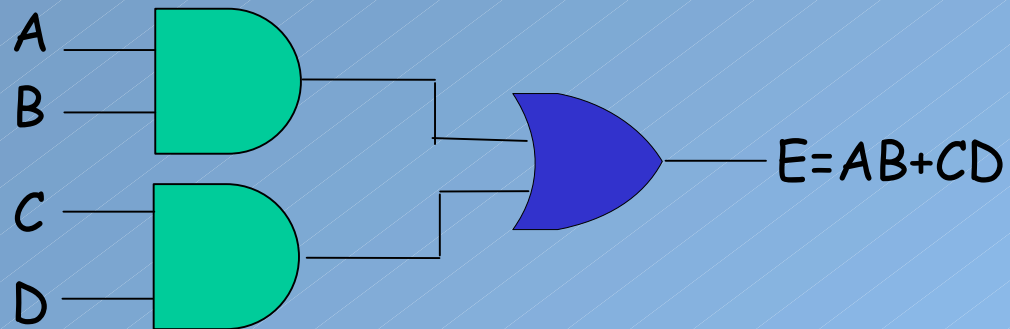
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NOR:  $C = (A+B)' = A'B'$

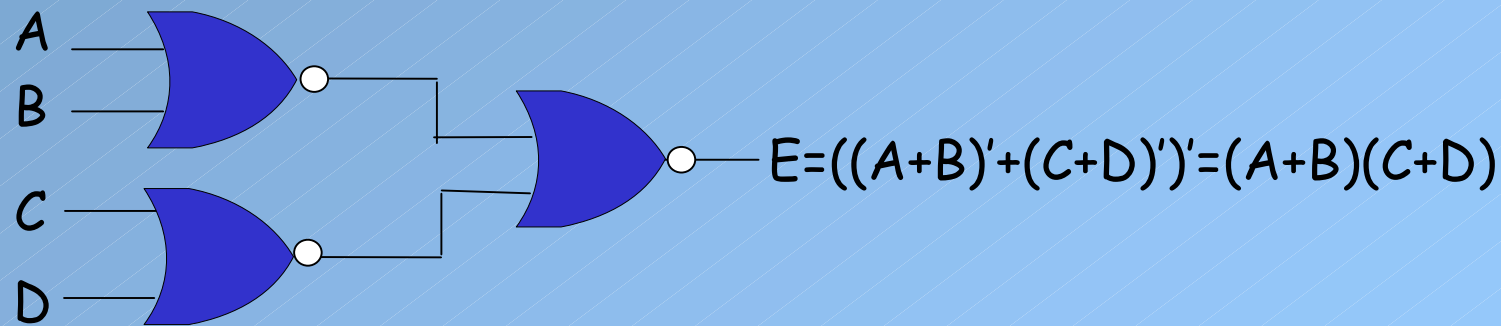
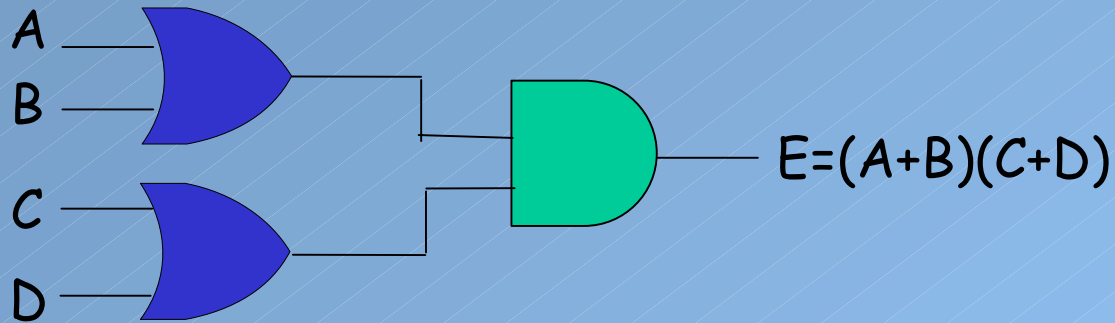
## Designing with NAND and NOR gates - 2

- Conversion of AND-OR network to NAND gates



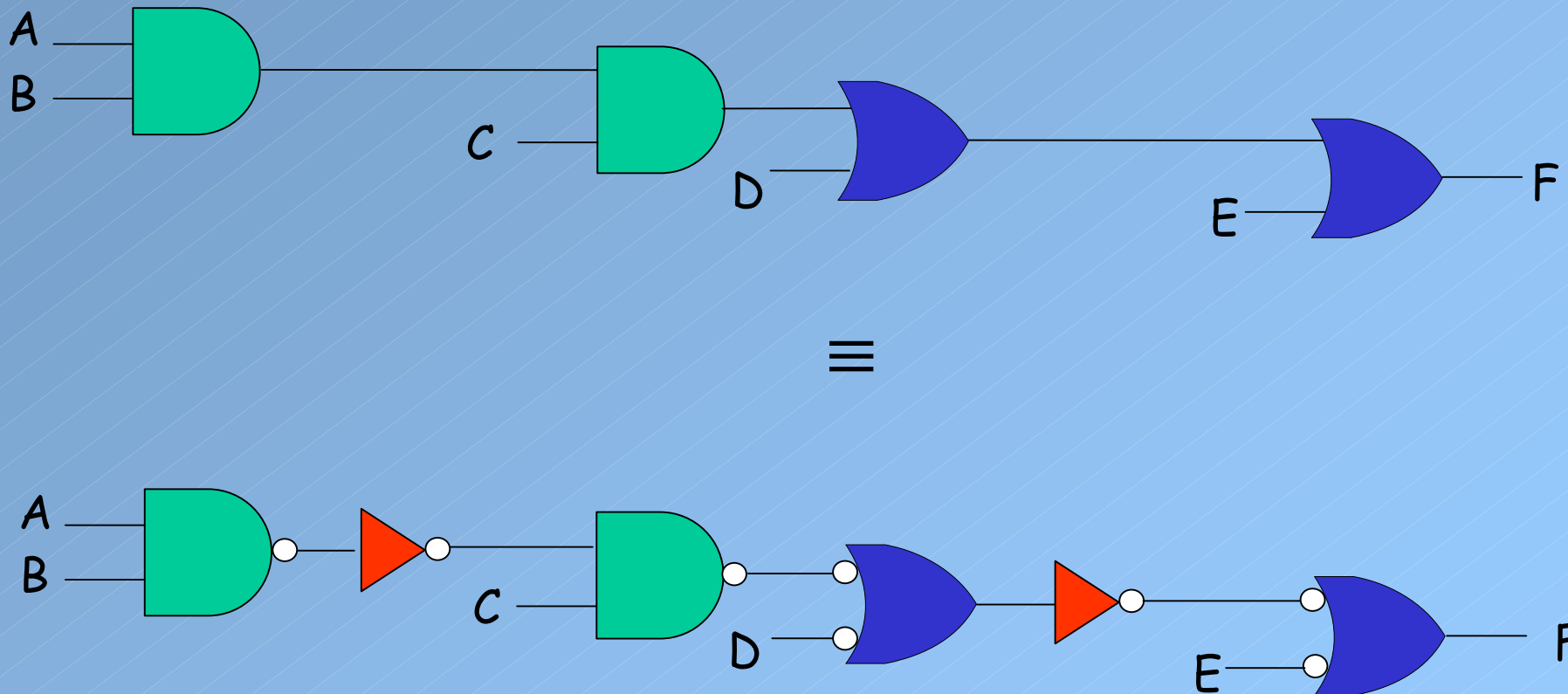
## Designing with NAND and NOR gates - 3

- Conversion of AND-OR Network to NOR gates



## *Designing with NAND and NOR gates - 4*

- Conversion of AND-OR network to NAND gates

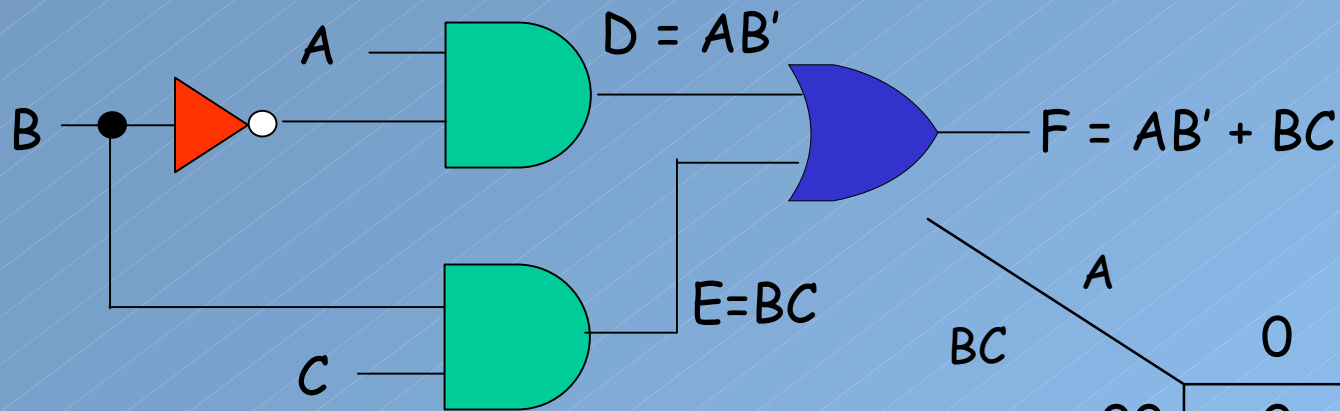


# *Hazards in Combinational Networks - 1*

- Unwanted switching transients
  - Occur when different paths from input to output have different propagation delays
  - *1-hazard*: network output may momentarily go to 0 when it should remain a constant 1
  - *0-hazard*: network output may momentarily go to 1 when it should remain a constant 0
  - *Dynamic-hazard*: when the output is supposed to change (1-0 or 0-1), the output may change three times (e.g. 1-0-1-0)

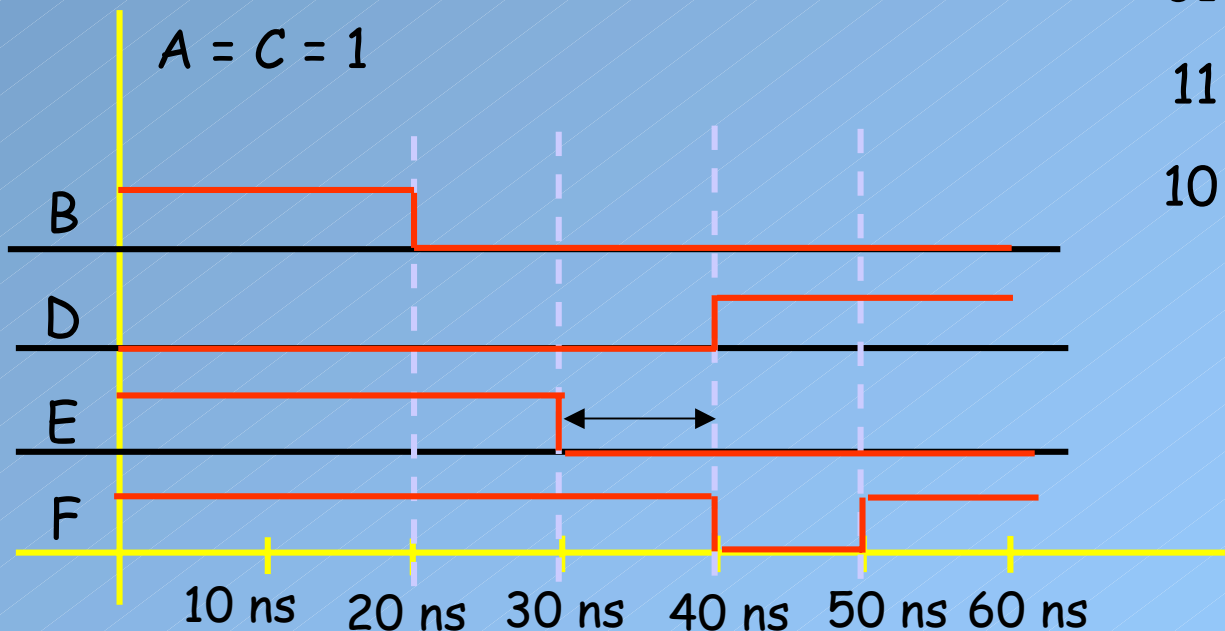


# Hazards in Combinational Networks -2



Each gate has 10 ns propagation delay!

$A = C = 1$

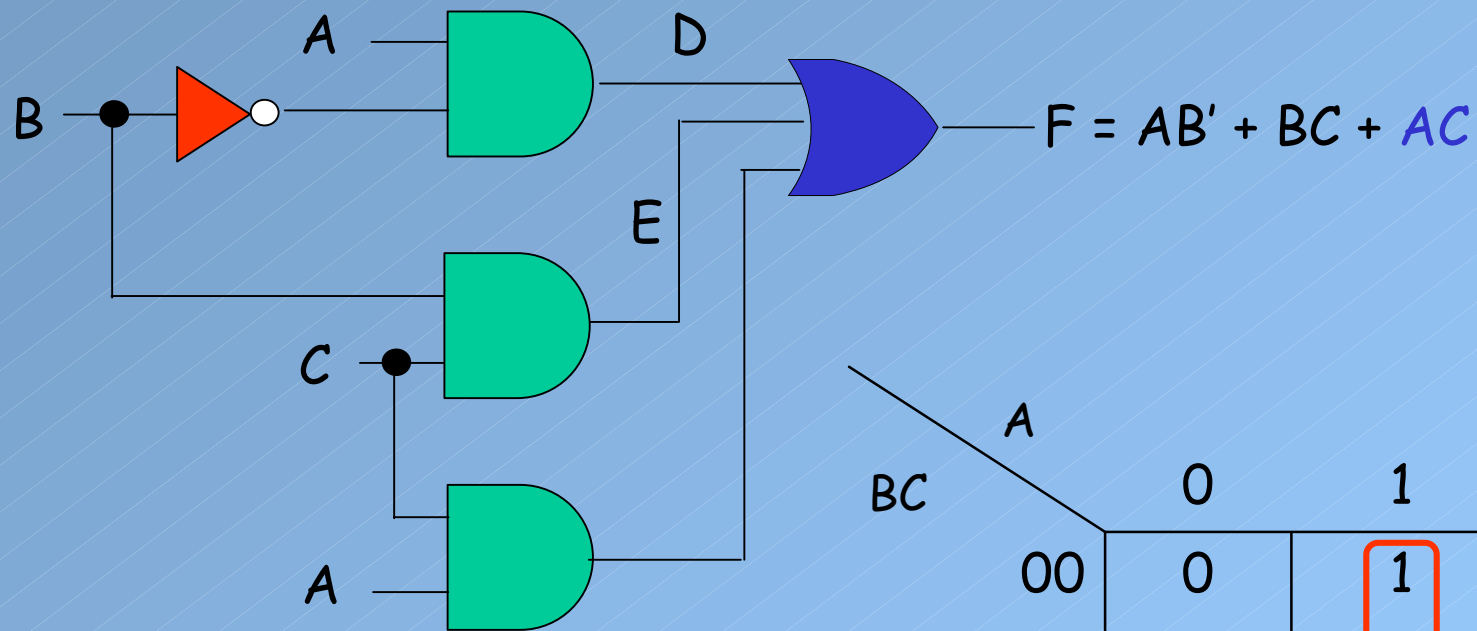


		A	
		0	1
BC	00	0	1
	01	0	1
	11	1	1
	10	0	0

1-hazard

# Hazards in Combinational Networks -3

- Remedy



		A	
		0	1
BC	00	0	1
	01	0	1
	11	1	1
	10	0	0

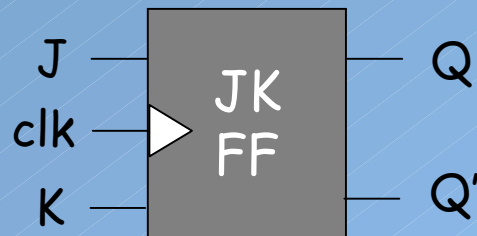
# Flip-Flops

## D Flip-Flop



D	Q	Q <sup>+</sup>
0	0	0
0	1	0
1	0	1
1	1	1

## Clocked JK Flip-Flop

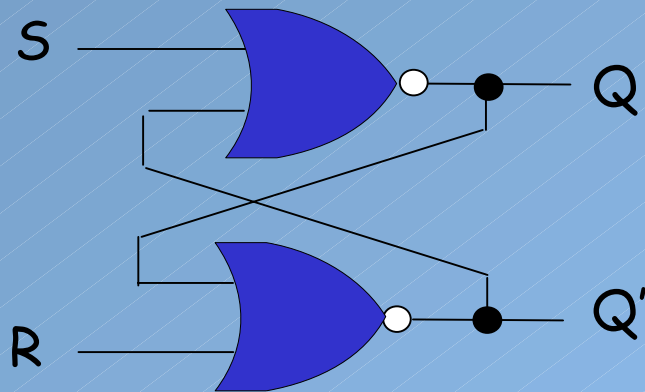


$$Q = JQ' + K'Q$$

J	K	Q	Q <sup>+</sup>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

# *Latches - 1*

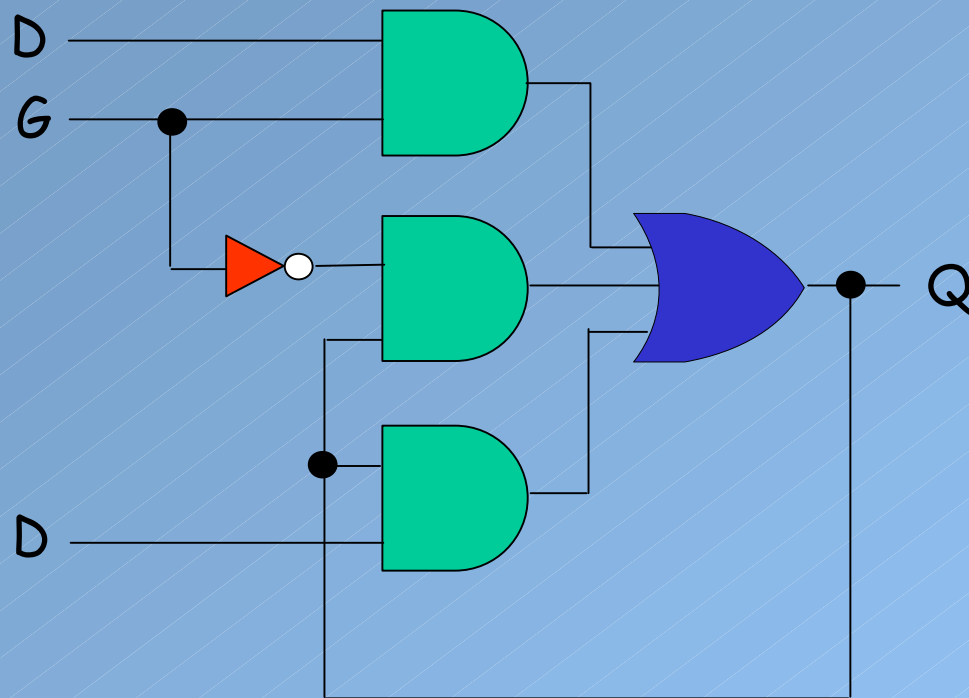
## S-R Latch



S	R	Q	Q <sup>+</sup>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	-
1	1	1	-

# Latches - 2

## Gated D Latch

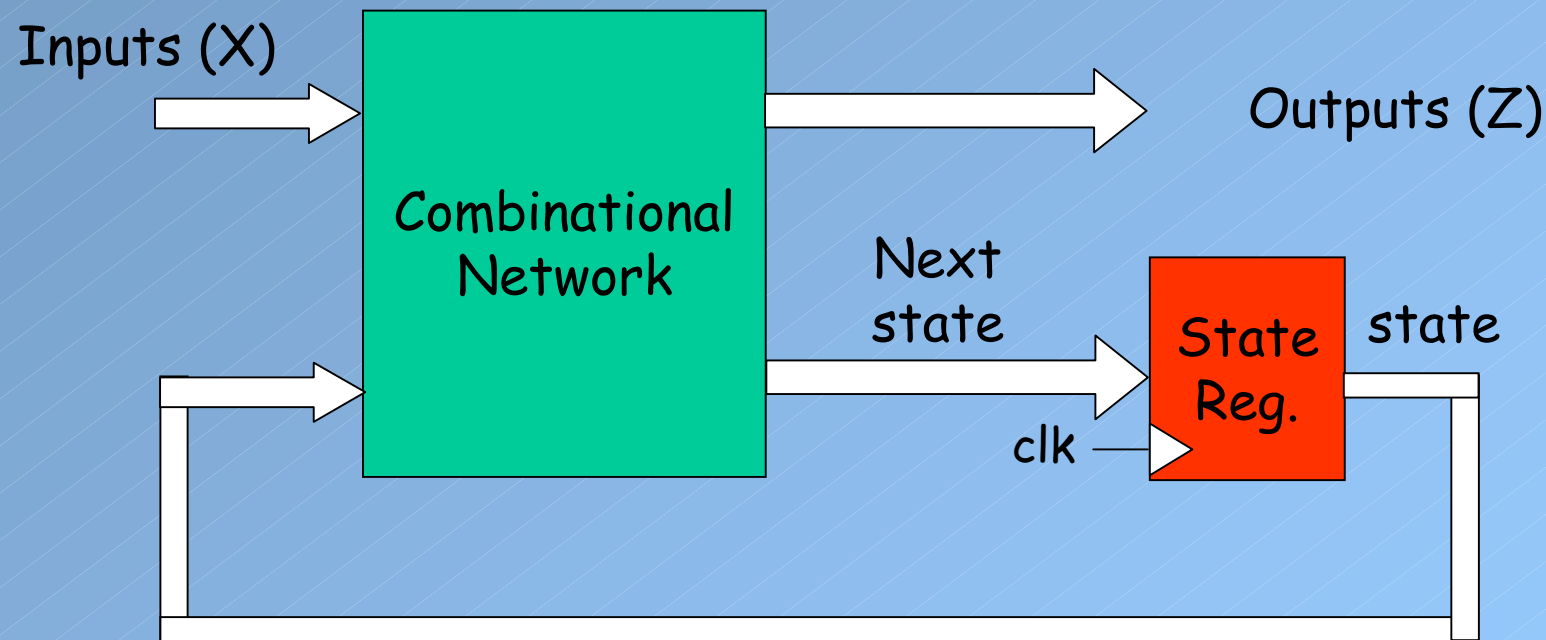


$G$	$D$	$Q$	$Q^+$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$Q^+ = DG + G'Q + DQ$$

# *Mealy Sequential Network Design*

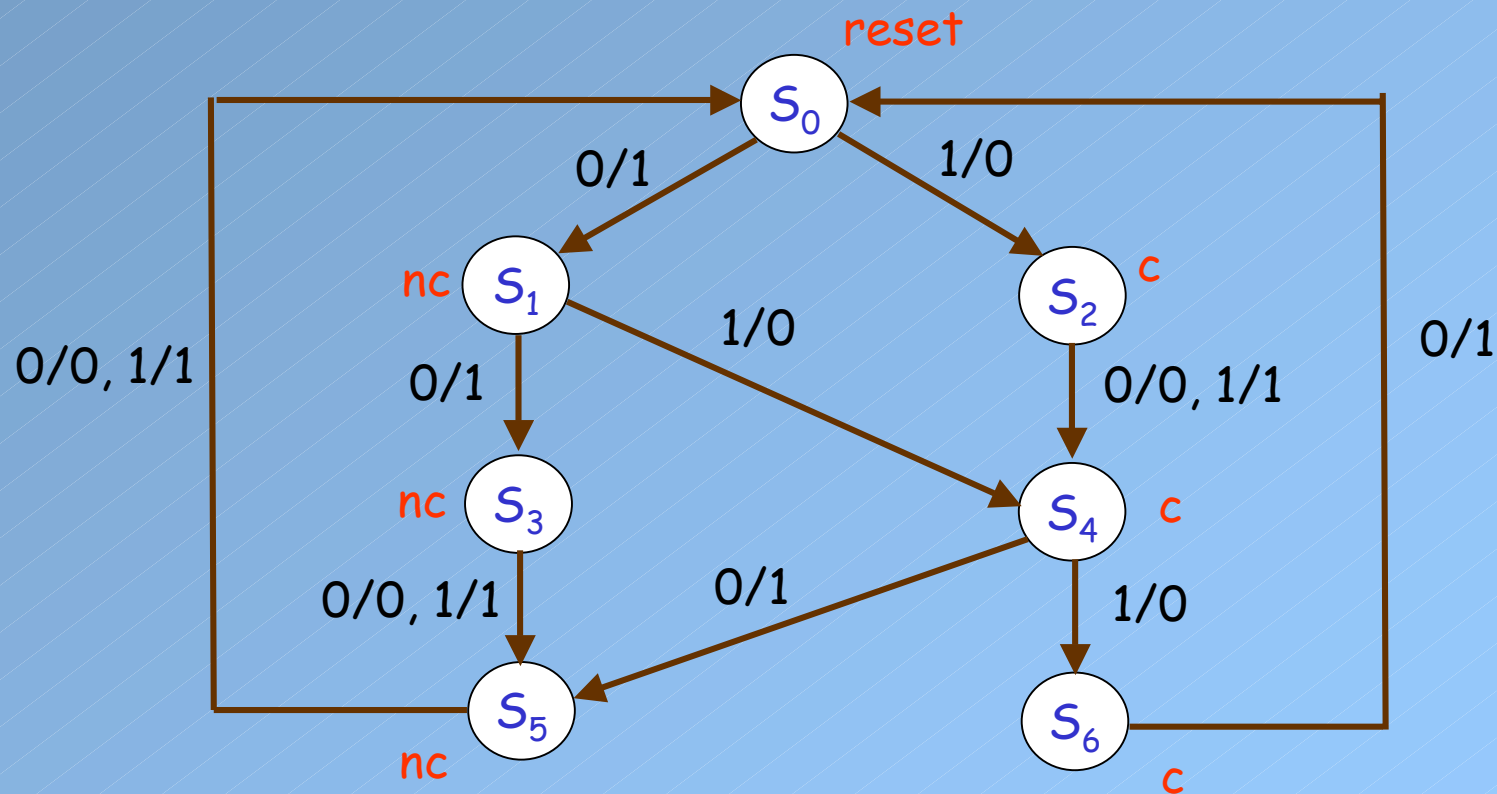
## General Model of Mealy Sequential Design



Outputs depend on both present state and present input

# Mealy Machine – Example (1)

- Problem: code conversion
  - Convert a BCD digit to another code by adding 3.
  - For example,  $(1001)_2 + (0011)_2 = (1100)_2$
  - Serial input/output, one bit at a time starting LSB.



## Mealy Machine – Example (2)

- State table

Present State	Next State		Output	
	input=0	input=1	input=0	input=1
$S_0$	$S_1$	$S_2$	1	0
$S_1$	$S_3$	$S_4$	1	0
$S_2$	$S_4$	$S_4$	0	1
$S_3$	$S_5$	$S_5$	0	1
$S_4$	$S_5$	$S_6$	1	0
$S_5$	$S_0$	$S_0$	0	1
$S_6$	$S_0$	-	1	-

- Seven states → three flip-flops
- Challenge: state assignment to FF so that logic is the simplest



# Mealy Machine – Example (3)

- State Assignments

$Q_2 Q_3 \backslash Q_1$		0	1
00		$S_0$	$S_1$
01		0	$S_2$
11		$S_5$	$S_3$
10		$S_6$	$S_4$

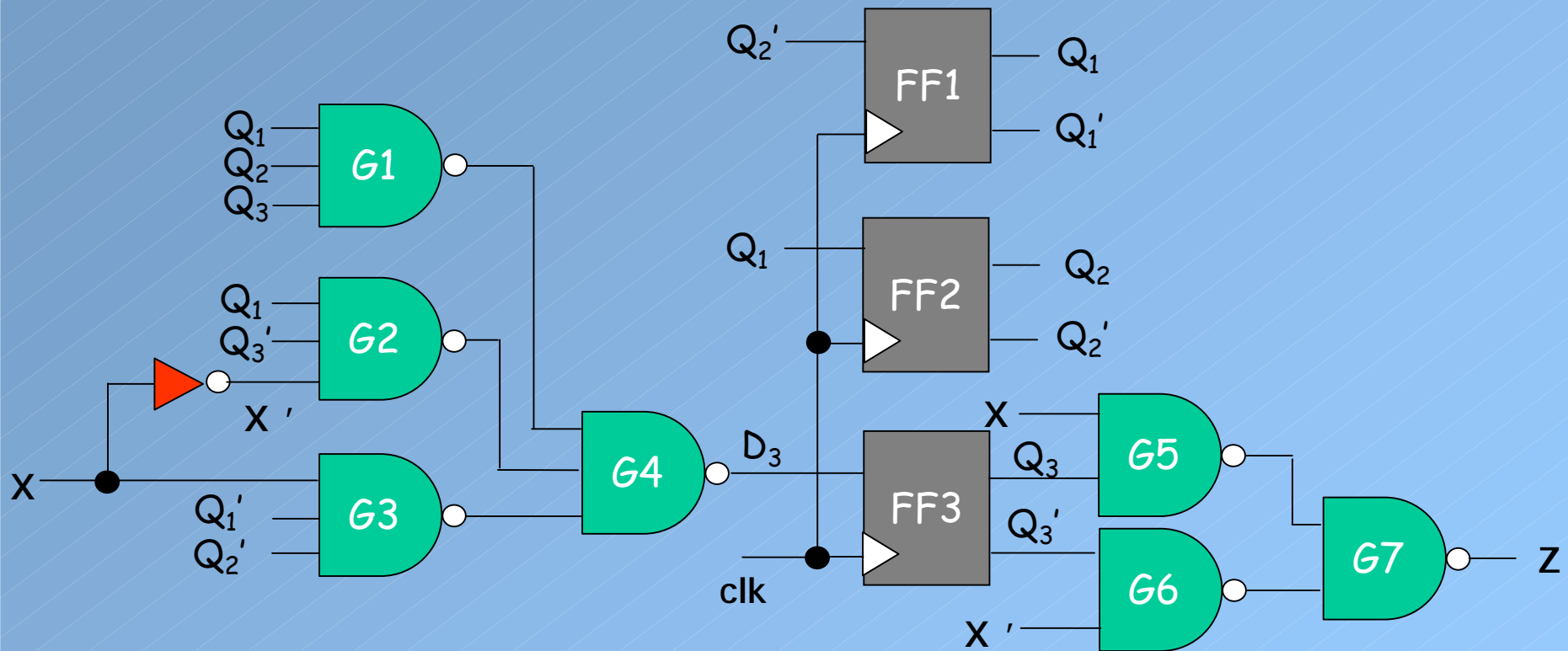
Assignment map

$Q_1 Q_2 Q_3$	$Q_1^+ Q_2^+ Q_3^+$		$Z$	
	$X=0$	$X=1$	$X=0$	$X=1$
000	100	101	1	0
100	111	110	1	0
101	110	110	0	1
111	011	011	0	1
110	011	010	1	0
011	000	000	0	1
010	000	xxx	1	x
001	xxx	xxx	x	x

Transition Table

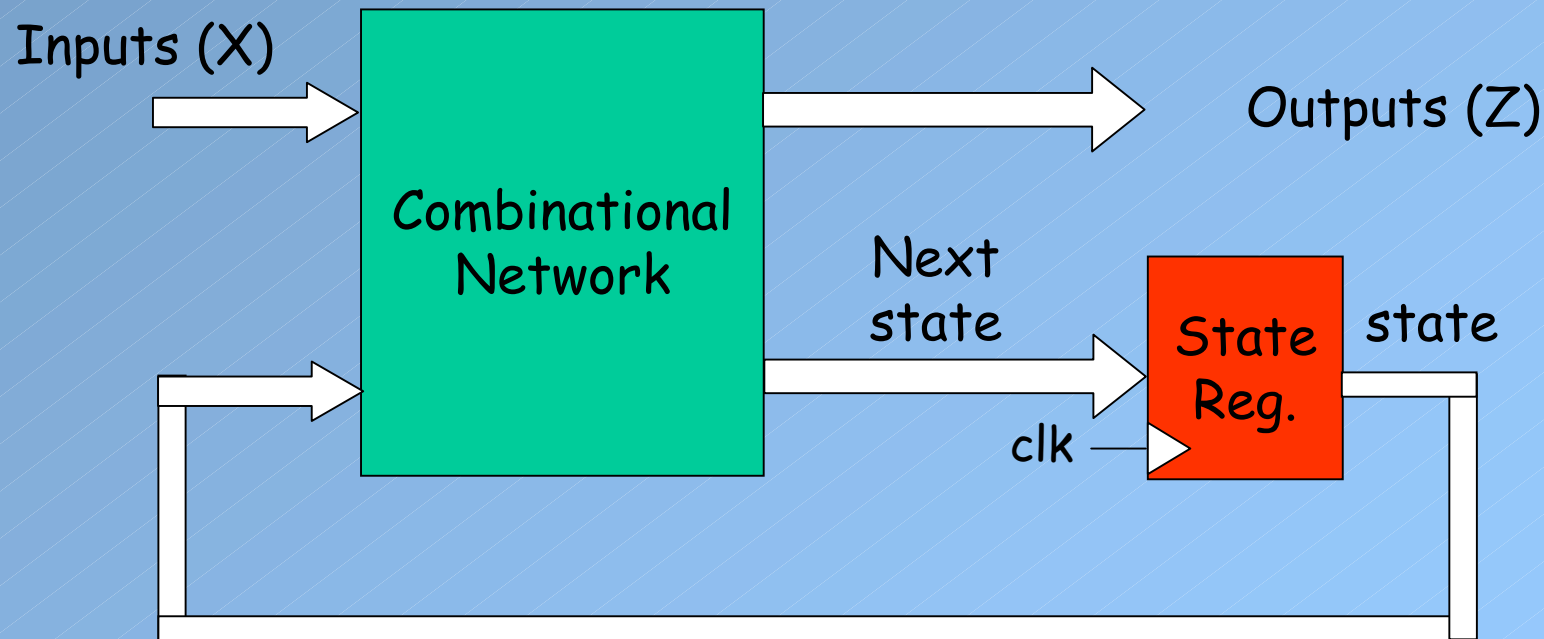
# Mealy Machine – Example (4)

- Realization of Code Converter



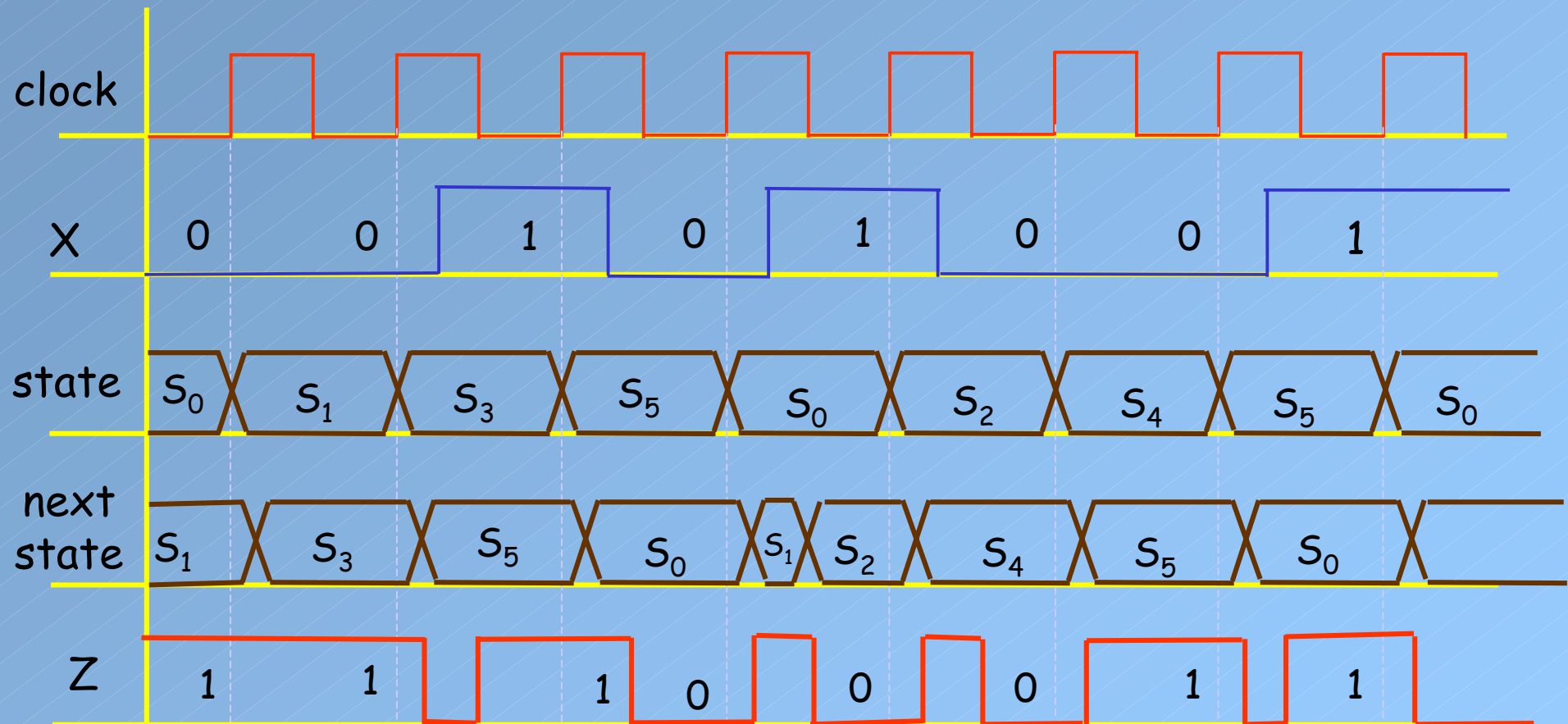
# Moore Machine

- The output is only function of the present state, independent of the input.
- Input determines the next state.



# Sequential Network Timing

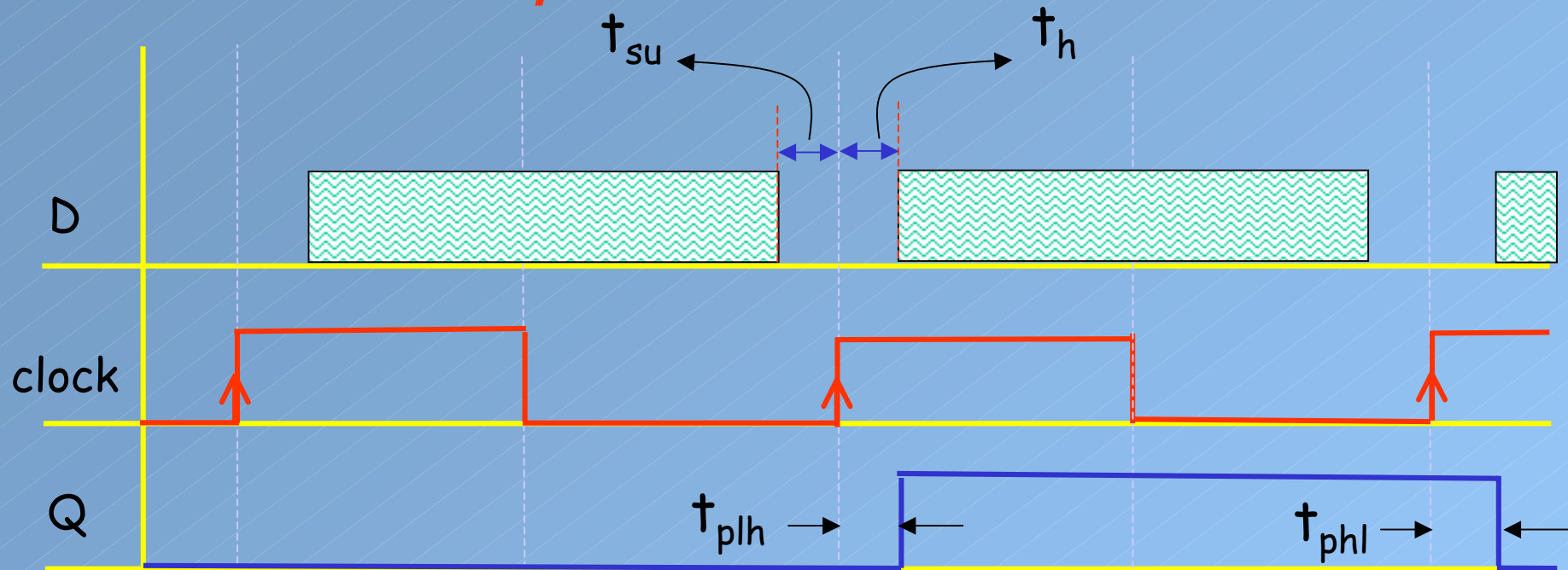
- Timing diagram for Code Converter



# *Setup and Hold Times 1*

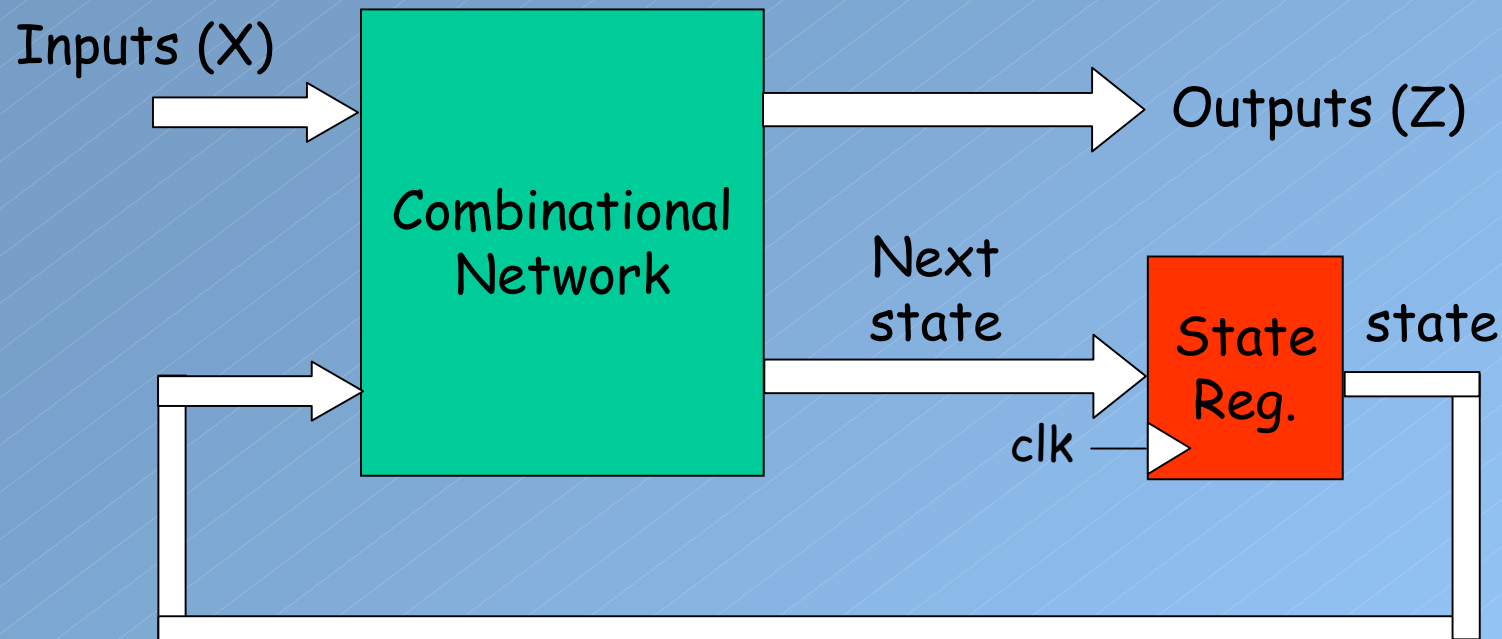
- For an ideal D flip-flop
  - If the D input changes at exactly the same time as the active edge of the clock, flip-flop operate correctly
- In reality
  - The D input must be stable for a certain amount of time before the active edge of the clock; setup time
  - Furthermore, D must be stable for a certain amount of time after the active edge of the clock; hold time

## Setup and Hold Times 2



- In shaded interval above D may be changed
- Otherwise, it cannot be determined whether the flip-flop will change state
- Even worse, the flip-flop may malfunction and output a short pulse and even go into oscillation

# Maximum Clock Frequency



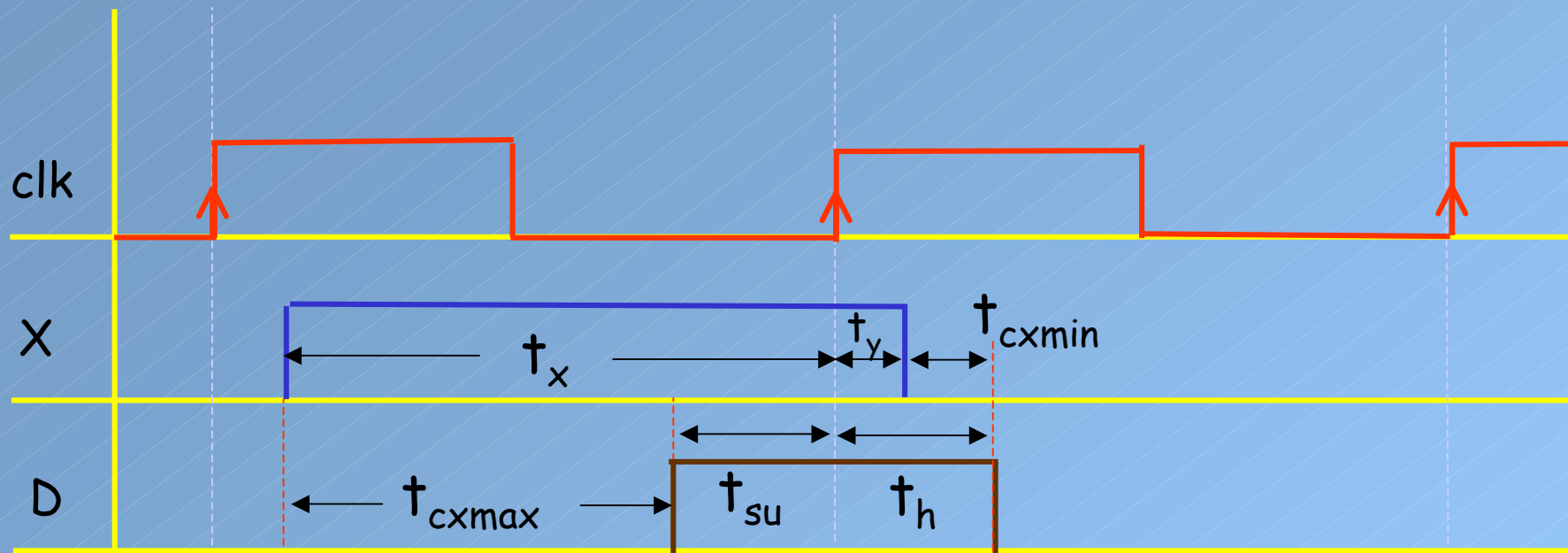
- $t_{ck}$ : clock period;  $t_{pmax} = \max(t_{plh}, t_{phl})$
- $t_{cmax}$ : maximum propagation delay through combinational network
- Then,  $t_{pmax} + t_{cmax} \leq t_{ck} - t_{su}$ .

# *Setup and Hold-time Violations (1)*

- Hold-time violation
  - If the change in Q fed back through the combinational network and caused D to change too soon after the clock edge.
  - $t_{pmin} + t_{cmin} \geq t_h$  ( $t_{pmin} > t_h$  for normal flip-flops)
- Usually, setup or hold-time violation occurs if the input X to the network changes too close to the active edge of the clock.
  - We must make sure that an input change propagates to the flip-flop inputs such that setup time is satisfied before the active edge of the clock.



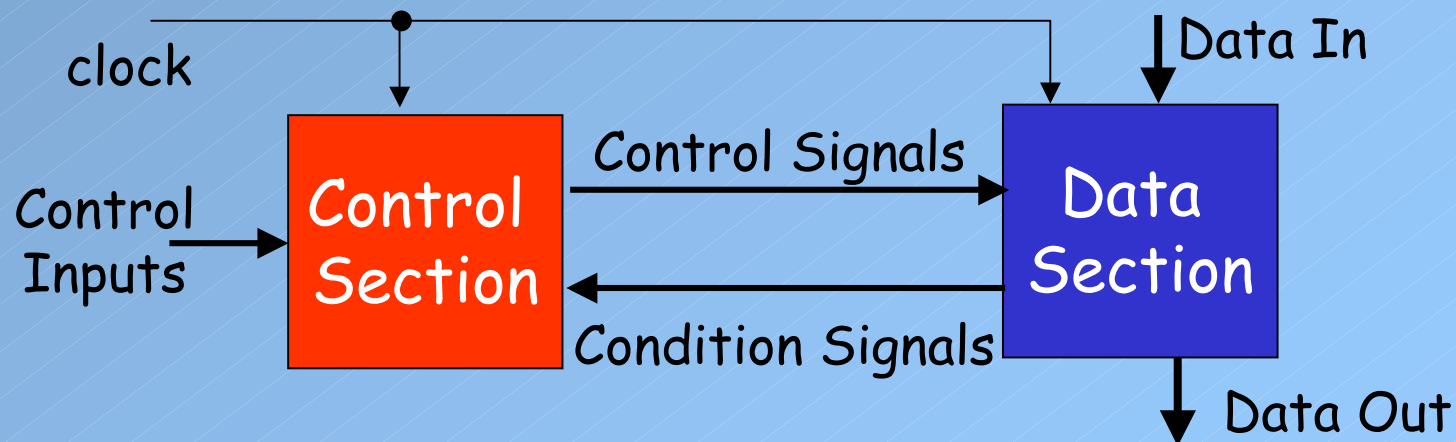
## Setup and Hold-time Violations (2)



- For setup time:  $t_x \geq t_{cxmax} + t_{su}$
- For hold-time:  $t_y \geq t_h - t_{cxmin}$  or  $(t_y + t_{cxmin} \geq t_h)$

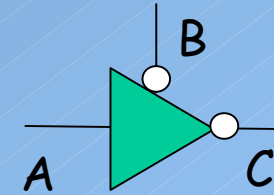
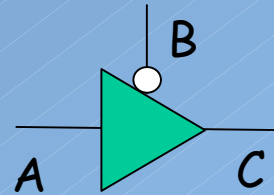
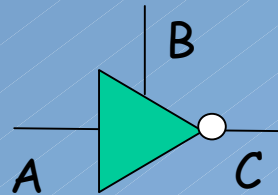
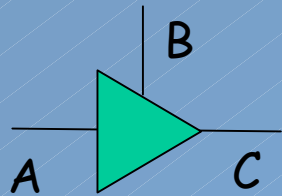
# *Synchronous Design*

- A clock is used
  - To synchronize the operation of all flip-flops, registers, and counters in the system
  - All state changes occur following the active edge of the clock
  - The clock period must be long enough so that all flip-flop and register inputs will have time to stabilize before the next active edge of the clock



# Tristate Logic and Buses

- Tristate buffers



B	A	C
0	0	Hi-Z
0	1	Hi-Z
1	0	0
1	1	1

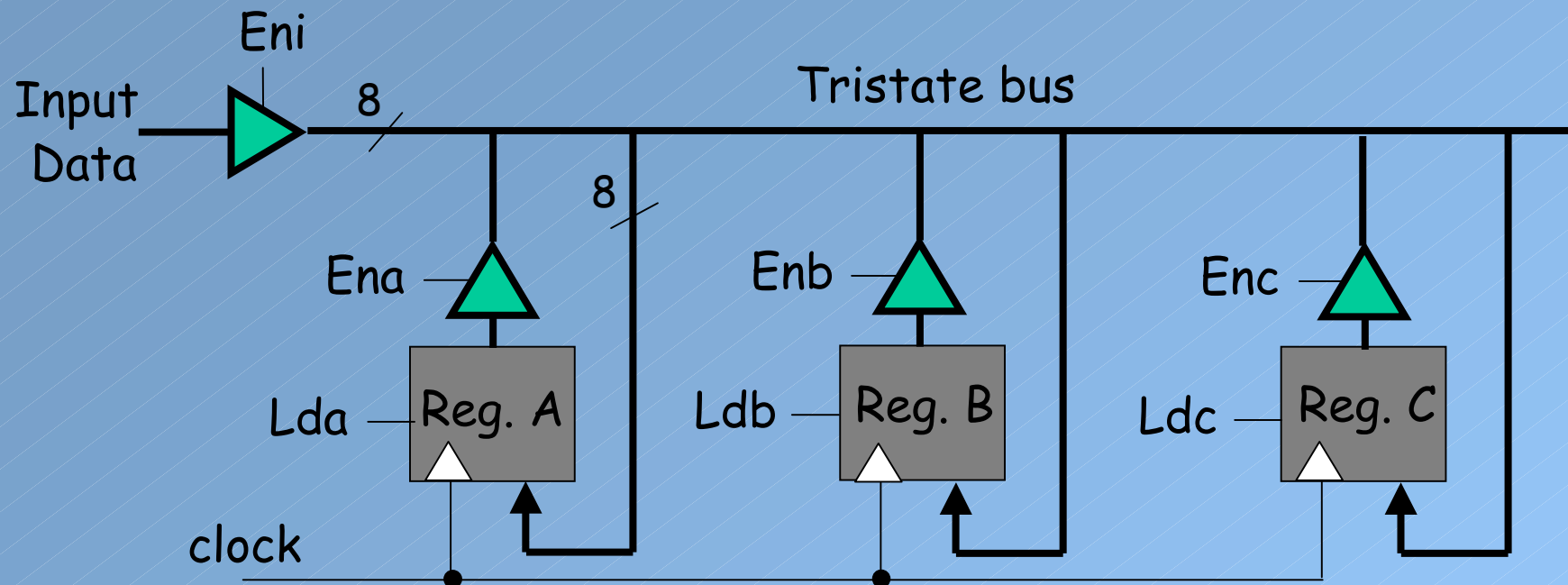
B	A	C
0	0	Hi-Z
0	1	Hi-Z
1	0	1
1	1	0

B	A	C
0	0	0
0	1	1
1	0	Hi-Z
1	1	Hi-Z

B	A	C
0	0	1
0	1	0
1	0	Hi-Z
1	1	Hi-Z

- Hi-Z : high-impedance which is equivalent to an open circuit.
- We use tristate buffers to connect the outputs of more than one gates or flip-flops.

# Tristate Bus



- If  $Enb = Ldc = 1$  (all others 0), then the data in register B will be copied into register C when the active edge of the clock occurs.
- If  $Eni = Lda = Ldc = 1$  then the input data will be loaded in registers A and C when the registers are clocked.