METAMATERIAL DESIGN VIA THE DENSITY METHOD[1]

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1 INTRODUCTION

Design of "new" electromagnetic devices such as antennas, frequency selective surfaces and filters with complex geometry and materials, possibly "new" composites and artificial dielectrics or meta-materials has become a realizable possibility. Some examples for these meta-materials are photonic and electromagnetic band gap materials for optical, millimeter-wave and microwave applications, anisotropic ferrites, ferroelectric ceramics and chiral materials [1-3]. The potential for improved designs and new devices using meta-materials prompt us to develop automated design tools which combine the capability to design for both geometrical and material topology.

Over the past several years, different design and optimization algorithms have been proposed and used for various electromagnetic applications [4-5]. Statistical search approaches, among them simulated annealing and genetic algorithms have already been used to achieve limited topology and material optimization [6, 7]. The majority of these dealt with geometry optimization where the topology and the material composition of the system is predefined. Thus, the design space obtained by these conventional methods is limited. Instead, this paper focuses on a design methodology capable of generating novel configurations which are designed for optimal shape topology as well as material distribution.

The Density/Solid Isotropic Material with Penalization (SIMP) method, based on the Optimal Material Distribution (OMD) approach, is the first topology/material optimization technique introduced for automated design of magnetic devices [8]. The OMD has a conceptually different design view than conventional methods. In the context of OMD, the universal parameterization of assigning each design cell (finite element/pixel) and a material property is employed. However, in the density method, the normalized density within each finite element is used as the design variable to formulate the topology/material optimization problem. The density method is simply a design approach enabling the modeling graded meta-material by assigning different density values from 0 to 1 in each cell to represent the material variation from cell to cell. Unlike conventional optimization methods, such as the boundary variation or 0/1 representation of the design domain, the density method arrives at the optimum topology by distributing the material in the form of a gray scale image using common gradient based mathematical programming techniques. This density method has been described in [8] for magnetic devices design, and has been quite successful in structural engineering [9]. In this paper we generalize and extent the application of the density method to design meta-materials for high frequency electromagnetic applications. The density design method is then used in conjunction with the Sequential Linear Programming (SLP) optimization algorithm to achieve optimal material distribution designs for dielectric substrates such as those used in constructing

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band-pass frequency selective volumes. As is the case with all numerical optimization approaches, the design loop involves two modules: the analysis and optimization module. The analysis module must incorporate a formulation which leads to a fast O(N) implementation and is also rigorous and allows for both material generality and material adaptability. For our case, the finite element-boundary integral procedures [10] are employed to allow for full freedom in material and geometry specification, and the fast spectral domain [10] algorithm used for the boundary integral implementation is O(N). On the other hand, the optimization algorithm must allow for fast convergence when dealing with thousands of design variables, and the density method in conjunction with the SLP is an ideal choice.

2 DESIGN APPROACH

The most straightforward image-based geometry representation is the "0-1" integer choice, where the design domain is represented by either a void or full solid material. However, this formulation is not well posed mathematically, but it can be well posed by incorporating microstructures into the extended design domain, and thus allow for material design with intermediate properties, i.e. materials which have graded properties. This is the essence of the density method in which material grading is achieved using a single variable, ρ, to represent the material properties of each finite element/pixel in the design domain. This material "law" models an artificial isotropic meta-material, mathematically manifested in the form of an empirical function, called the density function and represents the relationship between the material property (the relative permittivity ε_r) and ρ. The latter has the expression $\rho^n = (\varepsilon_i - \varepsilon_{air})/(\varepsilon_o - \varepsilon_{air})$ where ε_i and ε_o are the intermediate and original relative dielectric permitivities of the solid, respectively, and the power n is an empirical penalty parameter greater than 2 for convergence purposes [8]. This scalar variable ρ can be physically interpreted as the density of the material whose properties are in proportion to ρ^n. The proper material model is justified as the final design approaches either entirely solid (ρ=1 or $\varepsilon_i = \varepsilon_o$) or entirely void (ρ=0 or $\varepsilon_i = \varepsilon_{air}$) thru image processing/filtering techniques applied to the resulting topology obtained from the density design method. Using the density of each design cell/finite element as the design variable, a generalized layout problem can be formulated. Because the problem formulation for the density function is rather simple, it can be solved by optimizers such as SLP.

3 DESIGN EXAMPLE

As an example, a spectral filter in the form of a Frequency Selective Volume (FSV) is optimally designed subject to given specifications. FSVs are structures composed of cascaded planar periodic frequency selective surfaces (FSS) [11] sandwiched between multiple dielectric blocks/layers. Typically, the multilayers on which the FSS is printed are of constant composition. Here, our goal is to design the meta-material distribution within the dielectric volume of the layers to achieve the specified filter response. The desirable passband is 1-2.4 μm with a sharp transition in power transmission at the band-gap wavelength of 2.4 μm. The key requirement is that the filter transmits more than 90% up to 2.4 μm and less than 10% beyond that range. Initially, the printed FSS on a single layer substrate was designed to deliver the best possible performance. However, the achieved response using surface optimization required further refinement and the main goal here is to improve the response by modifying the material/volume distribution. Based on the design specifications and the density method, a general non-linear optimization problem is formulated as follows:

Minimize $$\sum_{1}^{2.4}\left(\left|\tau_{\lambda_i}-1\right|\right)^2+\sum_{2.4}^{10}\left(\left|\tau_{\lambda_i}-0.1\right|\right)^2$$

subject to the following constraints: $\sum_{1}^{N}\rho_i \cdot V_i - V_T \cdot \eta \le 0$, $0 \le \rho_i \le 1.0$, where τ_{λ_i} is the power transmission coefficient at wavelength λ_i, V_T is the total volume of the design domain, i.e. the prescribed volume of the dielectric blocks; η is the fraction of the design domain the designed material is limited to and ρ's are the densities of each finite element/pixel that make up the dielectric volume. A minimum of the objective function corresponds to a performance with a high transmission (τ~1.0) for wavelengths 1-2.4 μm and a vanishing one (τ~0.1) for wavelengths beyond that range (2.4-10 μm). The optimization problem is solved iteratively via the SLP method, capable of handling a large number of design variables. The densities are the updated material properties in the automated process of the design iterations. The complete design procedure is shown schematically in Figure 1. Of particular importance is that the objective function and the corresponding sensitivity analysis, both depend on the electromagnetic fields and require fast and accurate simulation of the FSV structure.

The optimized meta-material topology obtained for the filter is a gray scale image for the dielectric material of the filter and is shown in Figure 2. That is, each cell/finite element is filled with a certain color associated with a specific range of dielectric permittivity. We note that solution convergence was achieved in only 35 iterations for the 3048 design variables. The resulting power transmission curves are shown in Figure 3 for both the initial and optimized designs. It is evident that the performance was significantly enhanced through optimization of the dielectric volume material. But the resulting gray scale image prompts us to use gradual penalization techniques and image processing/filtering to achieve a realizable/manufacturable discrete solid design and this will be discussed at the conference.

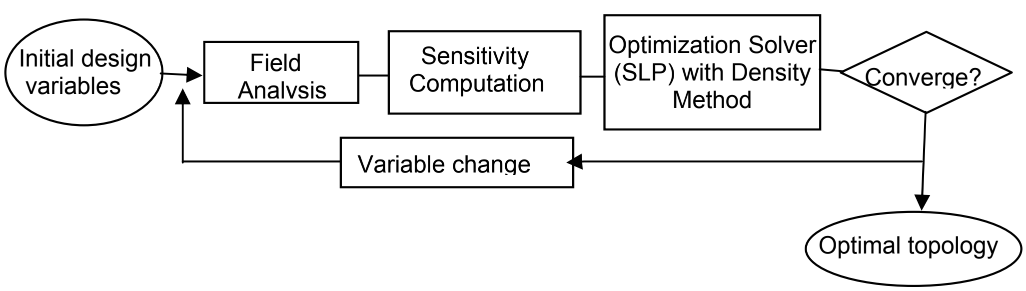

Figure 1: **Design Flowchart**

4 CONCLUSION

The capability of working on a fixed domain, representing shape without shape functions, and yielding an optimum topology, shape, and material distribution was demonstrated for electromagnetic applications. In this paper, we used density/SIMP method to design meta-materials for frequency selective surfaces/volumes. It was shown that significant improvement is attained by optimization of the material permittivity. The generality, low number of iterations and the simplicity of the demonstrated approach motivate application of the method for other microwave structures.

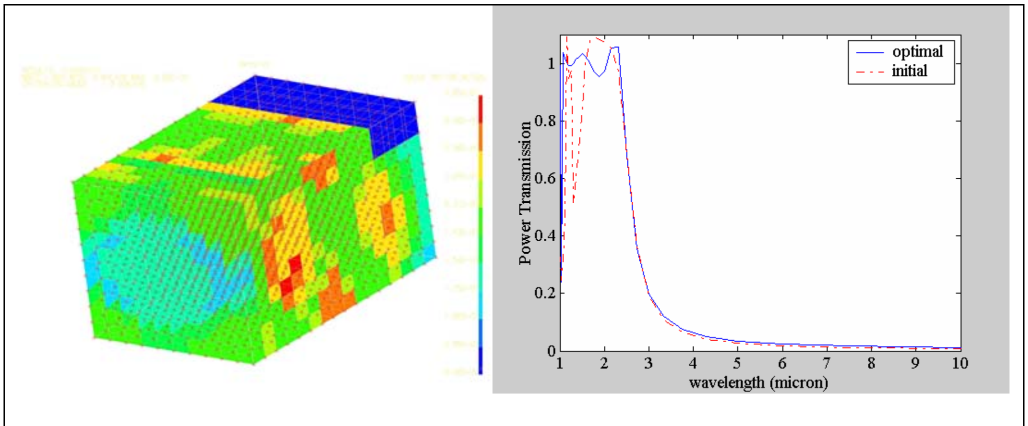

Figure 2. Resulting material distribution (left) and power transmission performance (right) for initial and optimal dielectric volume of the TPV filter design

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