Shape and Material Optimization for Bandwidth Improvement of Printed Antennas on High Contrast Substrates

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Abstract: It is well known that high contrast substrates, although suitable for miniaturization lead to smaller bandwidth. To overcome this limitation, a combined shape and material optimization technique is proposed to develop a miniature patch antenna with prespecified bandwidth performance. The employed design method is the Solid Isotropic Material with Penalization (SIMP) technique formulated as a general non-linear optimization problem. As part of the solution technique an adjoint sensitivity analysis and the Sequential Linear Programming Method (SLP) are adapted. A key aspect of the proposed design method is the integration of optimization tools with a fast simulator based on the finite element-boundary integral method. This allowed for inhomogeneous material modeling and design to increase the bandwidth of a fixed size patch antenna on a high contrast substrate having a dielectric constant of $\boldsymbol{e}_r = 100$.

Keywords: Topology Optimization, Material Design, Patch Antenna, Maximum Bandwidth, Miniaturization

Introduction

Design optimization for electromagnetic applications has traditionally been a process based on the creativity, past experience and knowledge of the designer. It has thus focused mostly on shape design or geometry of the device for performance improvement [1-3]. A more general design method is therefore likely to yield better overall performance. Such an approach draws from a broader class of design solutions as compared to conventional design methods and is capable of achieving designs that may yield much higher performance. Recent advances in fast and rigorous full wave simulators and the concurrent availability inexpensive manufacturing techniques for intricate shape and composite materials provides the opportunity to revolutionize traditional design optimization processes to topology and material optimization. This is expected to generate "new" electromagnetic devices that exhibit new properties and significantly better performance. So far, there are very few examples in the literature on topology optimization of electrical devices [4-5] and the majority of these dealt with problem specific, restricted or semi-analytic tools for magneto-static applications. That is, the subject of topology and material optimization is a fresh approach for developing novel RF device designs.

Two critical issues have to be addressed in developing novel designs. First, we need a general mathematical framework to conduct rigorous analysis of composite materials without imposing any geometric and material restrictions. Second, a versatile design method is required to find the best geometric configuration and material composition of the device. In this paper, the Solid Isotropic Material with Penalization Method (SIMP) [6] is integrated with fast hybrid finite-element boundary integral simulations [7] to develop full three-dimensional antenna designs addressing these two limitations. SIMP has been accepted as a design tool in mechanical engineering because 1) it allows for shape and material design without a-priori information on the initial shape or topology, 2) it incorporates a general non-linear optimization methods based on well defined optimization algorithms such as SLP [8], 3) employs a simple continuous function to relate the actual material property to the introduced density variable (and is hence well posed), and 4) updates the design variables thru the sensitivity analysis using the adjoint variable method which is efficient and permits full interface with the FE-BI electromagnetic solver.

In this paper, we introduce the mathematical framework for optimizing the material distribution of dielectric substrates using the SIMP method.. As an example, the SIMP method is extended here to design the material distribution of a dielectric substrate of a patch antenna subject to pre-specified bandwidth and miniaturization criteria. To our knowledge, this is the first ever integration of combined shape and material optimization for performance improvement of RF applications. A key element of this extension is the use of the latest fast algorithms recently introduced in the FE-BI formulation. For design optimization, the Sequential Linear Programming and an exact sensitivity analysis based on the solution of the adjoint problem [4] is employed. The sensitivity analysis is crucial to integrating the solver with the SLP optimizer.

The organization of the paper is as follows. First, the EM analysis tool is briefly introduced and subsequently the design procedure is outlined. This section starts with a discussion on the SIMP method and continues with the definition of the optimization model and its solution procedure.. To demonstrate the capability of the proposed method, 3D material design example for patch antenna radiation is employed to achieve 250% bandwidth improvement over the traditional substrates.

EM Analysis Tool

The proposed design optimization method is based on the integration of the optimization tool with a hybrid finite-element boundary integral simulator [7]. Application of hybrid methods to infinite periodic structures provides full 3-D modeling flexibility and allows for designing arbitrary geometrical and material details. By virtue of the finite-element method, the simulator is suitable for complex structures such as those involving inhomogeneous dielectrics, resistive patches, conducting patches and blocks, feed probes, impedance loads, etc. This makes the simulator an ideal candidate for generalized, yet efficient optimization loops.

The simulator employs the FE method to model a unit cell representing the doubly periodic array, whereas the BI provides for a rigorous mesh truncation at the top and bottom surfaces of the discretized unit cell. A key aspect of the periodic array model is the use of periodic boundary conditions (PBC's) to reduce the computational domain down to a single unit cell, thus significantly speeding up analysis and reducing memory resources. More specifically, a fast integral equation algorithm is used for an efficient evaluation of the boundary integral termination referred to as fast spectral domain algorithm (FSDA). The FSDA avoids explicit generation of the usual fully populated method of moments matrix. Instead, at each iteration, the actual current distribution is summed up in the spectral domain, and the spectral Floquet mode

series (for the BI) is carried out only once per testing function. Thus, for a fixed number of Floquet modes, the overall analysis method has O(N) memory demand and CPU complexity. Accurate results have already been obtained for scattering and radiation by cavities, slots, multilayer patch antennas and frequency selective surfaces, demonstrating the method's capability [7].

The conventional implementation of the hybrid FE/BI method for doubly periodic arrays leads to a linear algebraic system of the following form:

$$[A]\{E\} + [Z]\{E\} = \{f\} \qquad (1)$$

The A matrix is sparse and is associated with the FE portion of the hybrid method. Contributions of dielectric blocks or volumes and resistive cards or metallic edges in the unit cell are embedded in the [A] matrix. The [Z] matrix is associated with the edges on the top and bottom surfaces of the discretized unit cell and is fully populated. The right hand side vector {f} represents, as usual, excitations in the FE volume or BI apertures.

Design Procedure

Design Method

Topology optimization methods are general design methods used to obtain simultaneously the best geometric and topological configuration in terms of geometry, physical dimensions, connectivity of boundaries and material implants. They have reached a level of maturity and are being applied successfully to many industrial problems for almost 20 years [9]. Recently, some applications have also appeared in the electromagnetics, the majority of which are restricted to specific magneto-static applications [4-5]. For our design problem, we employ the topology optimization method based on the SIMP method to design miniature patch antennas with broadband behavior. The proposed method is aimed at designing the inhomogeneous structure of the dielectric substrate on which the patch is printed. Patch shape can alos be used for further bandwidth improvements but is not considered here since the focus is on material design.

SIMP is very attractive to the engineering community because of its simplicity and efficiency. It basically synthesizes the device starting from any arbitrary topology. A key aspect of the design method is that any device, not known a priori, is represented by specifying the material properties at every point of the fixed design domain. For electromagnetic applications, these are the permittivity and permeability of the dielectric material and conductivity/ resistance of the metallic patches, etc. In practice, to specify the material properties in the design region, the design space is discretized into material cells/finite elements. Actually, the most straightforward image-based geometry representation is the "0/1" integer choice, where the design domain is represented by either a void or a filled/solid material and this was adopted in [3]. However, this formulation is not well-posed mathematically [6]. It can be well-posed by allowing for the design of materials with intermediate properties; that is, materials having graded properties. This is the essence of the SIMP method in which material grading is achieved by introducing a single density variable, r, and relating it to the actual material property of each finite element thru a continuous functional relationship. A suitable interpolation for the permittivity (and possibly resistance of a metallic patch would be):

$$r = (e_{int} - e_{air})/(e_{orig} - e_{air})^{1/n} \qquad (2)$$

where n is a penalization factor; e_{int} and e_{orig} are intermediate and original solid material permittivity, respectively. As n increases, intermediate values for the permittivity are less likely to occur,

hence the term penalization for intermediate material. The on/off nature of the problem has been avoided through the introduction of the normalized density with r=0 corresponding to a void (air with e_{air}), r=1 to solid (original material e_{orig}) and 0< r <1 to a graded intermediate dielectric material (e_{int}). Moreover, this parameterization allows for the formulation of the problem in a general non-linear optimization framework. The goal is to arrive at the optimum distribution of material (densities) such that a certain performance merit of a device is optimized subject to certain design constraints. The problem formulation will be discussed in the next section.

Optimization Model

For our design problem, the goal is to determine the shape of the patch and its material distribution (substrate under the patch) subject to pre-specified bandwidth and miniaturization requirements. The first step is to reduce the design goals to a mathematical optimization model consisting of a mathematical cost function subject to constraints. An appropriate model for the corresponding topology optimization problem employing the SIMP method would be to find the design variables r to minimize the cost function:

$$f(r(e)) \qquad (3)$$

Subject to a volume constraint:

$$\sum_{i=1}^{NFE} \rho_i \cdot V_i \leq V^* \qquad (4)$$

and side constraints:

$$0 < \rho_{min} \leq \rho_t \leq \rho_{max}, \quad i=1,..NFE \qquad (5)$$

A possible cost function to maximize the return loss bandwidth would correspond to a minimization of the highest return loss among sampled frequency points N_{freq} as [1]:

$$f(r) = \min\left[\max(|s_{11}|_j)\right] \ j = 1,..N_{freq} \qquad (6)$$

The volume constraint is basically imposed to limit material usage. That is, a maximum volume V* of the material is allowed within the design domain. The actual material is comprised of the density r_i and volume V_i of each design cell in the FE domain. The necessity of the constraint is well known in terms of optimization convergence purposes: As the volume constraint is active, i.e. satisfied at the end of the design, it aids in achieving a black and white material composition. Physically, it can be viewed as a constraint that allows the comparison between the initial material (dielectric) substrate and the final optimized one. The final design has a broadband behavior with respect to the initial design but not through the reduction of its permittivity value. The side constraints are needed to allow for material usage within prescribed limits of the available materials with r_{min} being the normalized lower bound vector and unity the normalized upper bound vector.

The above design problem (2)-(6) is easily recognized as a general non-linear optimization problem with usually several thousand variables. This makes the use of gradient-based optimization techniques such as Sequential Linear Programming (SLP) [8] a must for the solution of the optimization process. The SLP is described in the next section.

Optimization Routine

The iterative optimization scheme chosen here is the sequential linear programming (SLP) method employing the DSPLP package

in the SLATEC library [10] due to its well-known efficiency and reliability. Other intuitive routines such as the GA or SA would be impossible to use considering its CPU requirements for a real 3D design composed of many design cells. The essence of the SLP routine is to replace the objective function and constraints by a linear approximation obtained from a Taylor series expansion about the current design point at each iteration. The most critical aspect in doing so is to employ the gradients or the derivatives of the mathematical functions in the optimization model with respect to the design variables as derived in the next section. The linear programming sub-problem is then posed to find the optimal design changes from the current design point. It is of great importance to impose constraints for the design changes known as move limit bounds to ensure convergence. Typically, during one iteration, the design variables are allowed to change by 5-15% of their original values.

Sensitivity Analysis

The computation of the required derivatives of the objective function f with respect to the design variables is referred to as the sensitivity analysis and is of great importance for any gradient based optimization technique. We briefly discuss it here. For our design problem, the objective is a mathematical function in terms of the return loss $|s_{11}|$ at each frequency j defined as:

$$|s_{11}|_j = \frac{|Z_{in_j} - Z_0|}{|Z_{in_j} + Z_0|} \quad (dB) \tag{7}$$

where Z_{inj} is the input impedance at the feed location at frequency j and Z_0 is the reference impedance. In general terms, the objective function is a function of Z_{in} relying on the unknowns solved for via the FE simulator. More specifically,

$$|s_{11}| = f(E(\boldsymbol{e}), \varepsilon) \tag{8}$$

where e refers to the element dielectric permittivity, the material property of the device to be optimized. The real function is differentiated with respect to a complex variable by using an appropriate approximation [4] and the chain rule as:

$$\frac{d|s_{11}|}{d\boldsymbol{r}} = \left\{ \frac{\partial|s_{11}|}{\partial \operatorname{Re}(s_{11})} \operatorname{Re}\left[\frac{\partial s_{11}}{\partial \boldsymbol{e}}\right] + \frac{\partial|s_{11}|}{\partial \operatorname{Im}(s_{11})} \operatorname{Im}\left[\frac{\partial s_{11}}{\partial \boldsymbol{e}}\right] \right\} \tag{9}$$

where the chain rule is employed to determine the derivative of the complex return loss functional and yields:

$$\frac{\partial s_{11}}{\partial \boldsymbol{e}} = \frac{\partial s_{11}}{\partial E} \frac{\partial E}{\partial \boldsymbol{e}} \tag{10}$$

The derivative term of the edge unknowns E requires the differentiation of the previously discussed system matrix of the FE-BI formulation (1) with respect to the permittivity in each design cell as follows:

$$\frac{\partial[A]}{\partial \boldsymbol{e}}E + [A]\frac{\partial E}{\partial \boldsymbol{e}} = f \quad \Rightarrow \frac{\partial E}{\partial \boldsymbol{e}} = [A]^{-1}\left(\frac{\partial f}{\partial E} - \frac{\partial[A]}{\partial \boldsymbol{e}}E\right) \tag{11}$$

Substituting this into (10) results in:

$$\frac{\partial s_{11}}{\partial \boldsymbol{e}} = \frac{\partial s_{11}}{\partial E}[A]^{-1}\left(\frac{\partial f}{\partial E} - \frac{\partial[A]}{\partial \boldsymbol{e}}E\right) \tag{12}$$

Realizing that the first term requires a huge matrix inversion process, the solution of an adjoint problem to the original FE system seems to be more appropriate. More specifically, denoting the first term as $\boldsymbol{I}$ and taking its transform will result in:

$$\boldsymbol{I}^T = [A]^{-1}\left(\frac{\partial s_{11}}{\partial E}\right)^T \tag{13}$$

Due to the symmetric nature of matrix [A], (13) can be rewritten as:

$$[A]\{\boldsymbol{I}\}^T = \left\{\frac{\partial s_{11}}{\partial E}\right\}^T \tag{14}$$

The key part of the whole sensitivity analysis is based on the solution for the adjoint vector $\boldsymbol{I}^T$ using the original FE system equations with the RHS replaced by the term as above for each iteration only once. The final sensitivities in (9) for each design cell are obtained by substituting the adjoint variables in (12) and carrying out the outlined steps above. It is important to note that the term in the parantheses of (12) are carried out on the local element matrix level of each design cell the permittivity of differentiation refers to. This whole process allows for significant savings in the CPU requirement while maintaining the accuracy of analytical differentiation.

Design Algorithm

The algorithm for the proposed design cycle is shown in Fig. 1. The design cycle starts with the initialization of the design variables, which corresponds to an initial homogenous dielectric substrate with a certain permittivity value. Certain design parameters are also specified at this step but do not change during the design cycle. These are design parameters such as patch geometry and material characterization, like the dielectric block dimensions. Also, the feed location and amplitude and the frequency range of operation are specified. The next step is the discretization of the design domain into a large number of finite elements and the distribution of the available dielectric material throughout the domain. Consequently, the iterations start and at each iteration until convergence the following steps are executed as displayed: 1) Simulation of the device performance using the FE-BI solver and fed in data 2) Solution of the adjoint system equations of the original problem for the sensitivity analysis 3) Optimaztion of the material distribution of the dielectric material within the design domain using an SLP algorithm and 4) Updating the design variables (densities/permittivities of design cells) relying on the interpolation scheme of the SIMP design method. Convergence is achieved when the changes in the objective function value (hence the changes in the design variables) drop below a certain value like 10^{-3}.
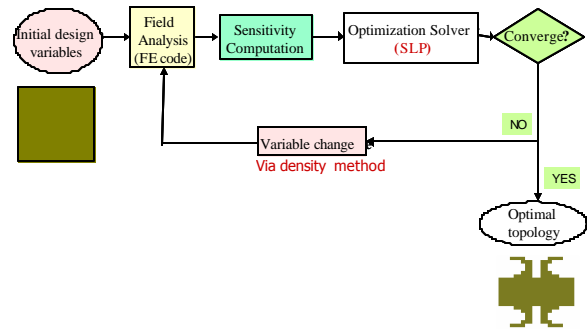

Fig. 1 Design Optimization Flowchart

Design Example

As is well known, microstrip patch antennas are attractive, low-weight, low-profile antennas which however suffer from low bandwidth. Moreover, its bandwidth is further reduced as the substrate dielectric constant is increased for miniaturization. In

this section, we demonstrate the capability of the outlined design method to develop a small patch antenna subject to pre-specified bandwidth criteria. The main goal is to improve the bandwidth performance of a chosen simple patch antenna by introducing a new (metamaterial) substrate texture whose properties are not found in nature.
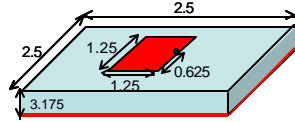

Fig. 2 Schematic of the patch antenna on the dielectric block substrate (dimensions are in cm).

The chosen geometry is a square patch fed with a probe/coax feed and the details of the whole structure are displayed in Fig. 2. The frequency range of interest is 1-2 GHz sampled over 21 frequency points. For this design, the volume was set to 70% with respect to the initial substrate dielectric permittivity to ensure better miniaturization for the designed substrate. The volume constraint is also necessary to avoid trivial bandwidth improvement via lower dielectric constant.
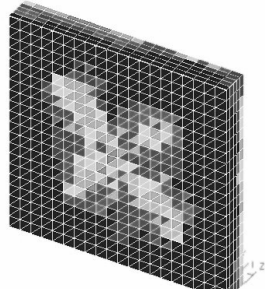

Fig. 3 Optimally designed metamaterial texture

The initial design using a homogenous substrate having e =42 resulted in a 5dB return loss bandwidth of 6.7% and we were not able to obtain the typical 10dB bandwidth because of the capacitive nature of the high contrast material. By pursuing the above discussed design procedures, with each design cell being updated via the SIMP method and the SLP routine, a heterogeneous design was obtained in 20 iterations. The converged material distribution is displayed in Fig. 3 as a 3D color coded block with each color pixel corresponding to a certain density/permittivity value. The corresponding return loss behavior of the optimized dielectric distribution is depicted in Fig. 4 and compared to the initial performance.
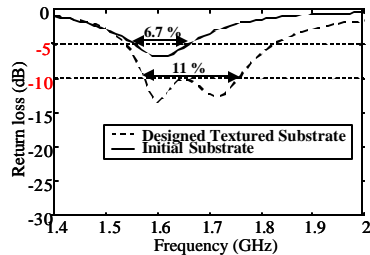

Fig. 4 Return loss behavior for initial and designed textured substrate

The computation time for the entire design process with a design domain of 4000 finite elements/design cells and 21 frequency points for each iteration was 17 hours on a Pentium 3 Processor.

Given the poor bandwidth at the starting point of the design, the attained bandwidth performance (with material design only) is truly remarkable. Further bandwidth improvements are possible via patch shape design and this will be discussed at the conference. It is obvious that the resulting graded material is not manufacturable. To be manufacturable, it must undergo certain post processing such as image processing or penalization techniques within the optimization cycle. Basically the former is a filtering process and the latter implies penalization of the intermediate permittivity (grey) values with a higher penalization factor n [6].

Conclusion

The aim of this paper was to demonstrate the capability of designing for shape and material using the SIMP method. This has been done by designing for variable material substrates to improve antenna performance and more specifically bandwidth. To achieve a 250% improvement in a fixed size patch antenna, the SIMP method has been extended along with the SLP routine for the solution. Key to the success of the optimization was the integration of optimization tool with fast, full wave FE-BI solver and the utilization of the sensitivity analysis. As demonstrated by the design example, by virtue of the generality and efficiency of the proposed method, there is great potential for improving antenna performance or other RF devices.

References

[1] J.M. Johnson and Y. Rahmat-Samii, "Genetic algorithms and method of moments (GA/MoM) for the design of integrated antennas," *IEEE Transactions on Antennas and Propagation*, vol. 47, no. 10, pp. 1606-1614, October 1999

[2] E. Michielssen, J. M. Sajer, S. Ranjithan, R. Mittra, "Design of light-weight, broad-band microwave absorbers using genetic algorithms," *IEEE Transactions on. Microwave Theory and Techniques*, vol. 41, no. 6/7, pp. 1024-1031, 1993.

[3] Z. Li, Y. E. Erdemli, J.L. Volakis and P.Y. Papalambros, "Design Optimization of Conformal Antennas by Integrating Stochastic Algorithms With the Hybrid Finite-Element Method," *IEEE Transactions on Antennas Propagaion,* vol. 50, no. 5, May 2002

[4] D. N. Dyck, D. A. Lowther and E. M. Freeman, "A method of Computing the Sensitivity in Electromagnetic Quantities to changes in Materials and Sources," vol. 30, no. 5, Sept. 1994

[5]J. Yoo and Noboru Kikuchi, "Topology Optimization in magnetic fields using the homogenization design method," *International Journal for Numerical Methods in Engineering*, vol. 48, pp. 1463-1479, 2000

[6] O. Sigmund, "Design of multi-physics actuators using topology optimization - Part I: One material structures", *Computer methods in applied mechanics and engineering*, vol. 190, pp. 6577-6604, 2001

[7] T. F. Eibert and J. L. Volakis, "Fast spectral domain algorithm forhybrid finite element / boundary integral modeling of doubly periodic structures," *IEEE Proceedings: Microwaves, Antennas and Propagation*, vol. 147, no. 5, pp. 329-334, October 2000

[8]S. S. Rao, *Engineering Optimization: Theory and Practice,* 3rd ed. New York, Wiley, 1996

[9]H. C. Gea, "Topology Optimization: A new microstructure-based design domain method," *Computers and Structures*, vol. 61, no.5, pp.781-788, 1996

[10]R. Hanson and K. Hiebert, "A sparse linear programming subprogram," *Technial Report SAND81-0297*, Sandia National Laboratories, 1981