Sabanci University FENS Faculty of Engineering and Natural Sciences

# High Sensitivity Rotation Sensing with Atom Interferometers using Aharonov-Bohm effect

SPIE 2006  Jan 26, 2006

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# INTRODUCTION

- Sagnac Effect → Rotation Sensing
- Optical Interferometers
- Atom interferometers
- Aharonov-Bohm Effect
- Atom Interferometer in a Faraday Cage
- Acceleration Sensitivity
- Conclusions

# SAGNAC EFFECT…

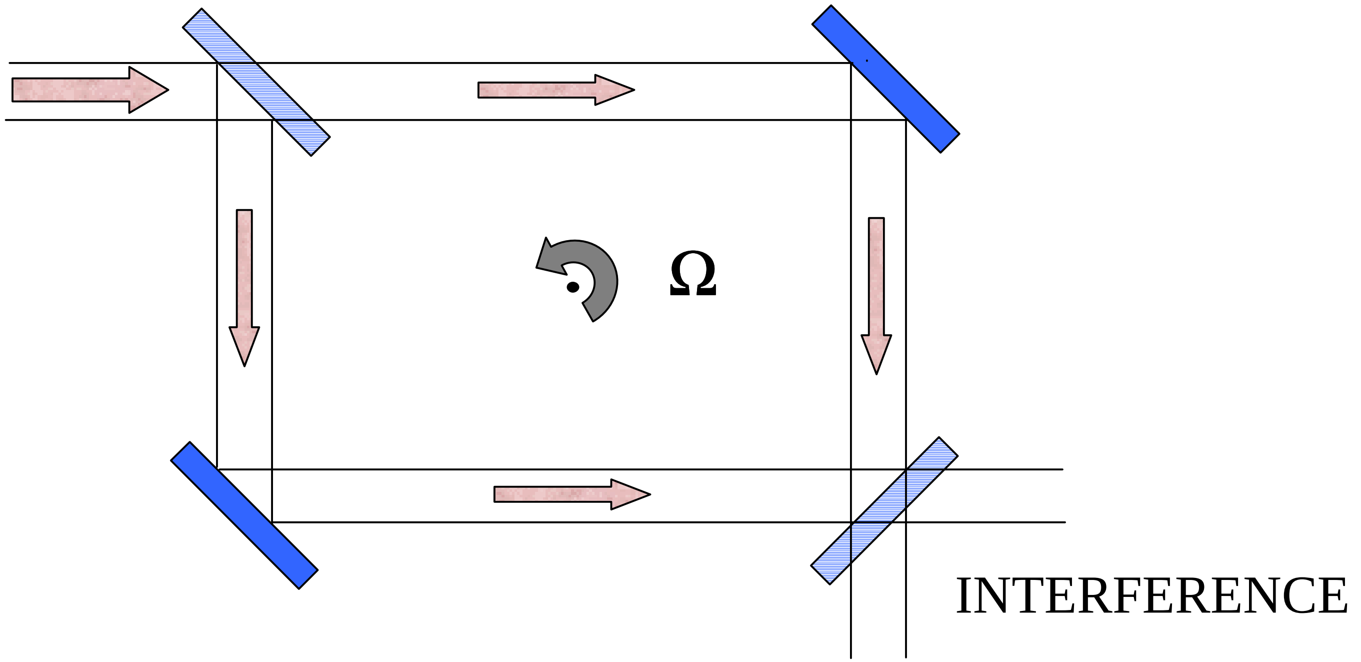
Sagnac effect is a rotationally induced phase shift between two paths of an interferometer…

Gyroscopes based on this effect measure a rotation rate relative to an inertial frame of reference…

Sensitive gyroscopes have applications in:

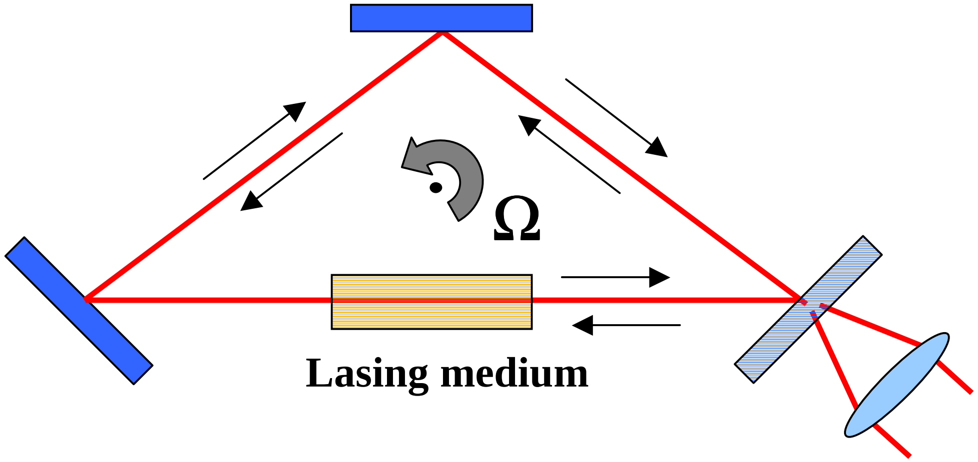
Navigation
Geophysics
General Relativity

# FREE SPACE OPTICAL INTERFEROMETER



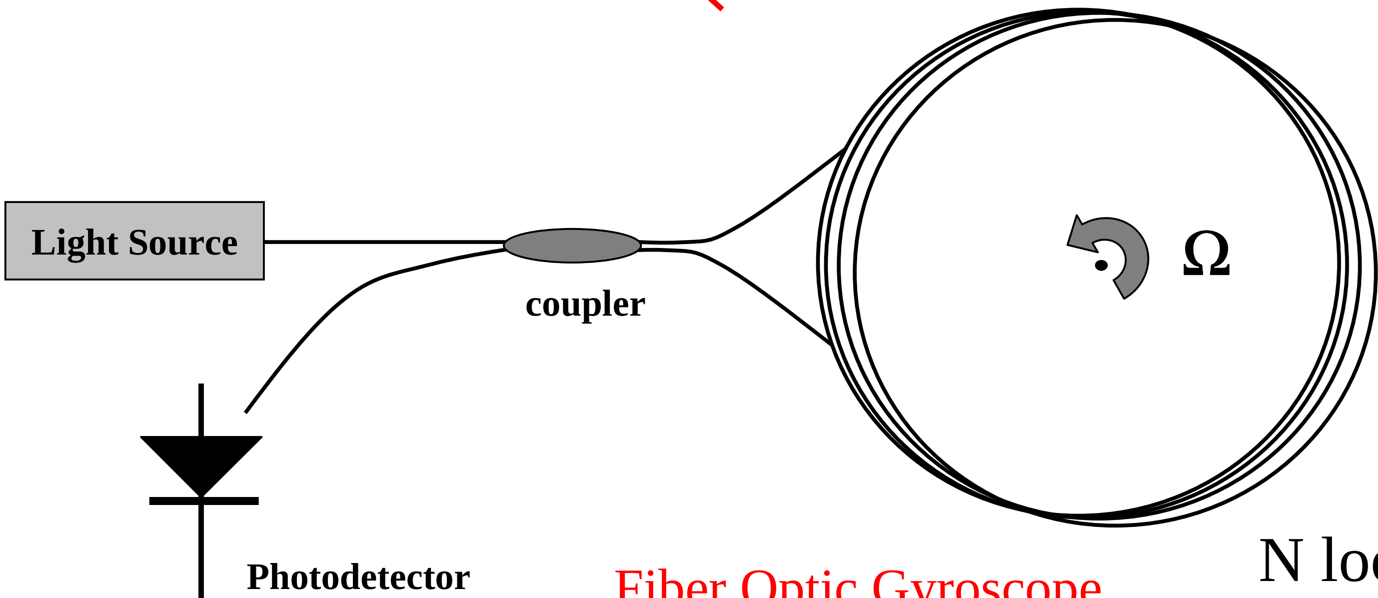

Free Space Optical Gyroscope

# RING LASER GYRO…FIBER OPTIC GYRO…




Most sensitive interferometer for rotation sensing…

Ring Laser Gyroscope




Fiber Optic Gyroscope

N loops of fiber

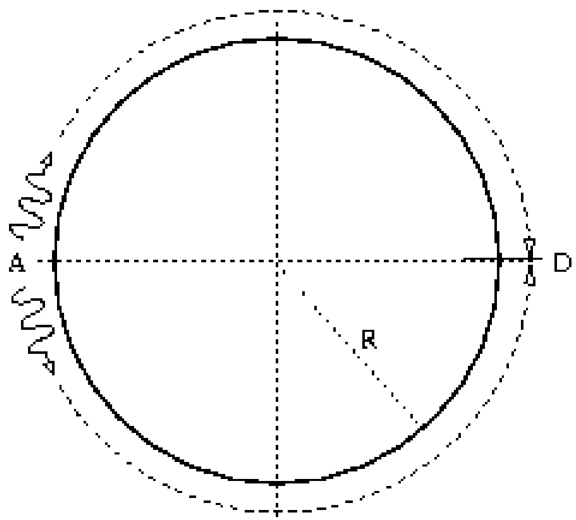
# SAGNAC PHASE SHIFT…

Assuming a circular interferometer in vacuum and motionless…counter-propagating waves reach to the detector in time $\boldsymbol{\tau = \pi R/c}$.

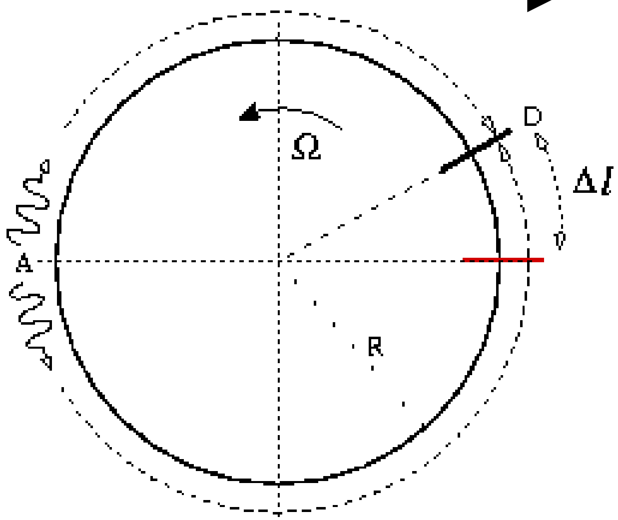
Now if the interferometer rotating at a rate of $\Omega$ rad/sec with respect to a motionless inertial frame of reference, observer will see that the detector moves $\boldsymbol{\Delta l = R\,\Omega\, t}$ during the flight time of the waves…

The counter-rotating wave will reach to the detector earlier with a time difference of $\boldsymbol{\Delta\tau = 2\,\Delta l/c}$, and for a wave of frequency $\boldsymbol{\omega}$ leads to a phase difference of $\boldsymbol{\Delta\Phi = \omega\,\Delta\tau}$. →

$$\Delta\Phi = \frac{4\pi\Omega\,.\,A}{\lambda c}$$




a)




b)

# SAGNAC PHASE SHIFT IN A DIELECTRIC…

A dielectric medium or waveguide does not change the Sagnac phase.

Sagnac phase shift in a solid medium is always given by the vacuum expression irrespective of the velocity (phase velocity) of the light…

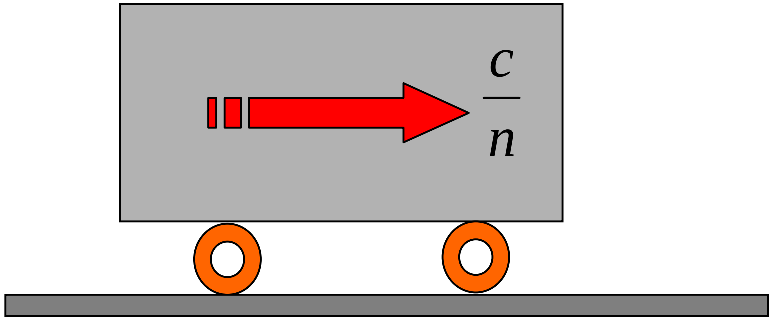
$$\Delta\Phi = \frac{4\pi\,\Omega\,.\,A}{\lambda c}$$

or →

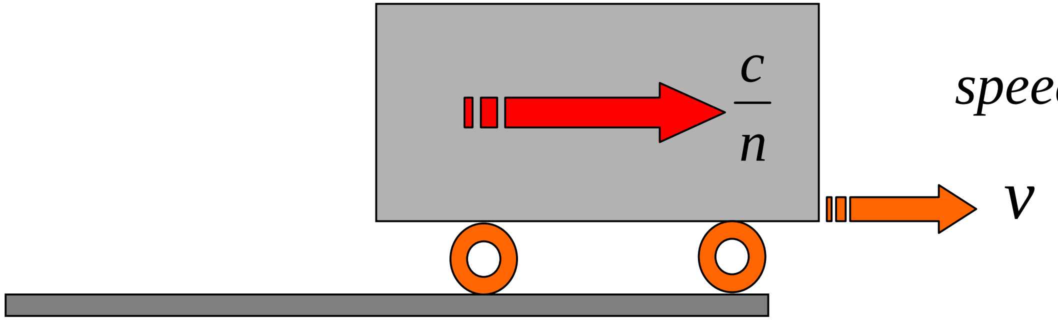
$$\Delta\Phi = \frac{4\pi\, f\,\Omega\,.\,A}{c^2}$$

# FRESNEL-FIZEAU DRAG

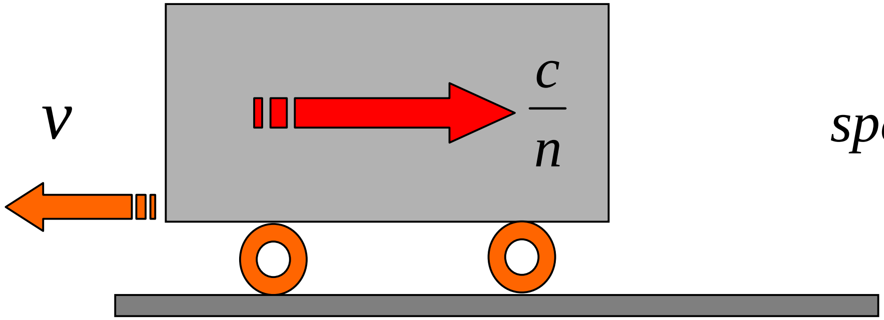
Light propagation in a moving dielectric…




$$speed\ of\ light = \frac{c}{n}$$




$$speed\ of\ light = \frac{c}{n} + \alpha v$$




$$speed\ of\ light = \frac{c}{n} - \alpha v$$

$$\alpha = (1 - \frac{1}{n^2})$$

Drag coefficient
Ignoring dispersion

# SAGNAC EFFECT WITH MATTER WAVES

$$\Delta\Phi = \frac{4\pi\,\Omega\,.\,A}{\lambda c}$$

by artificially replacing:

$$\lambda c = \lambda_m\, v_a$$

and by using the de Broglie relation:

$$p = \hbar\ k = \frac{h}{\lambda_m}$$

$$\Delta\Phi = \frac{4\pi}{h} m\ \Omega\,.\,A$$

Matter Wave Sagnac Phase Shift is also independent of velocity of the waves…

where $\lambda_m$ is the de Broglie wavelength, $v_a$ is the velocity of the atoms and $p = m\, v_a$.

# ADVANTAGE OF ATOM INTERFEROMETERS…

Why use atoms?

Because they are inherently much more sensitive than an optical interferometer:

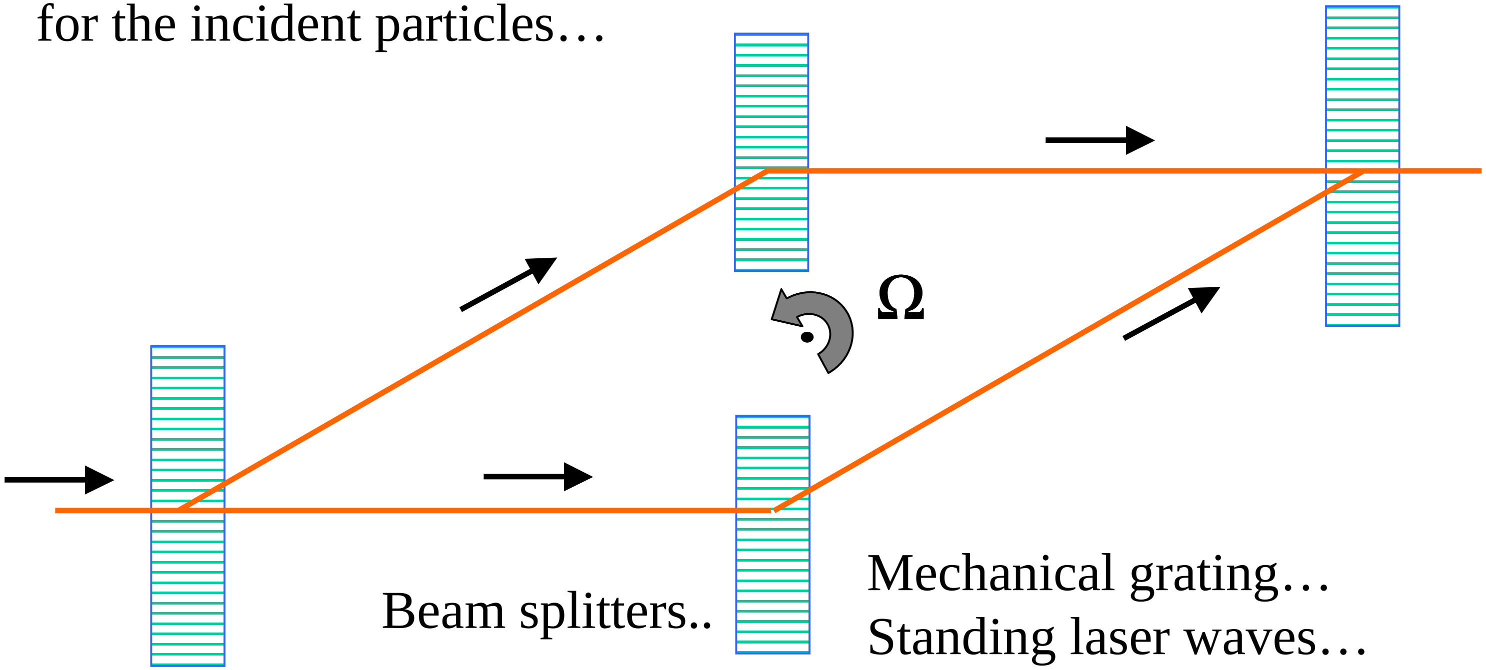
$$\frac{\lambda c}{\lambda_m v_a} \quad \text{or} \longrightarrow \quad \frac{m c^2}{\hbar \omega} \approx 10^{11}$$

(for Cesium Atoms it evaluates to $6\times10^{10}$ compared to a HeNe laser…)

To date: rotation sensitivity of atom interferometers reached to $\approx 6\times10^{-10}\ rad/\sec$ in 1 Hz bandwidth…which is as good as best ring laser gyros…

# TYPICAL ATOM INTERFEROMETER

The key element of most matter wave experiments is the diffractive beam splitters…ideally a scattering potential for the incident particles…




Mechanical grating…
Standing laser waves…
Two photon stimulated Raman transitions…two counter propagating laser beams with a frequency equal to hyperfine splitting of the ground states…

# AHARONOV-BOHM EFFECT

In 1954 Aharonov and Bohm published a paper: Significance of Electromagnetic Potentials in the Quantum Theory…., Phys. Rev. 115, p.485,1959…

Consider a charged particle inside a Faraday cage connected to an external generator which causes the potential change only as a function of time…

This will add a term to the Hamiltonian: $H = H_0 + V(t)$

$$H_0 \longrightarrow \Psi_0(x,t)$$

$$H \longrightarrow \Psi_0(x,t) \ \exp\left[ -i\frac{q}{\hbar}\int V(t)\,dt \right]$$

PHASE ONLY TERM!

# A-B EFFECT Continued…

$$H \longrightarrow \Psi_0(x,t)\ \exp\left[-i\frac{q}{\hbar}\int V(t)\,dt\right]$$

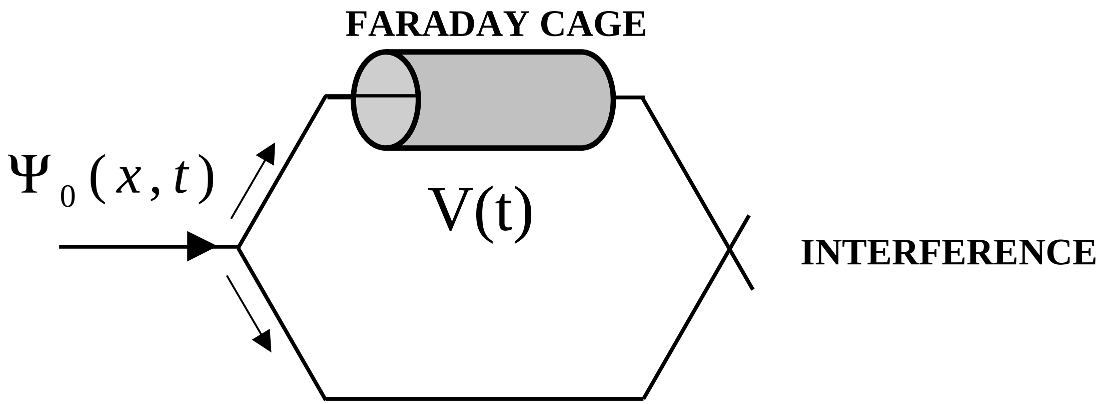
Since the new solution differs only by a phase factor…no change in the physical result! Particle motion is not affected at all!

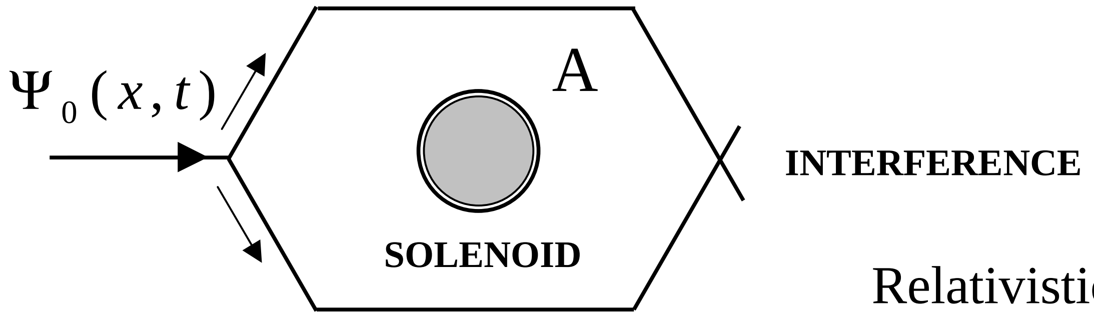
However if the particle beam is split into two parts and one part is passed through the potential region…an interference of the two parts will result…

Thus, there is a physical effect of the potentials even though there is no actual force is ever acted on the particles!

Similar situation exists for a vector potential A…Outside an ideal solenoid B field is zero…However a particle encircling such a solenoid attains a phase shift!

# A-B EFFECT Continued…



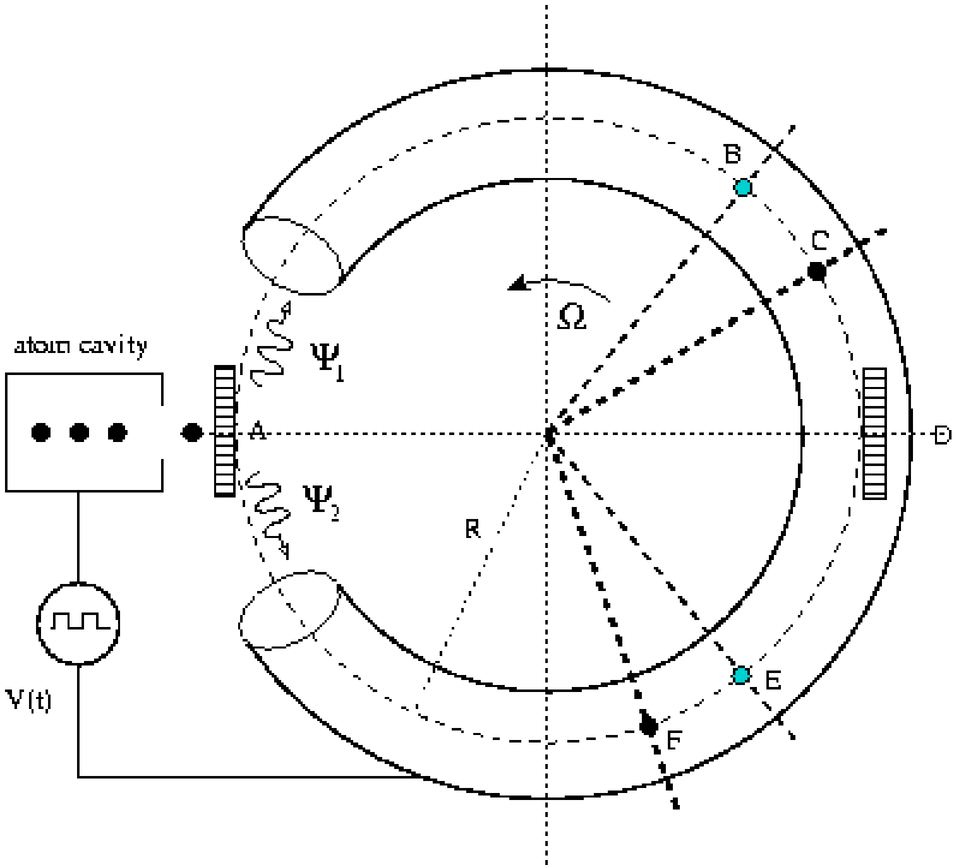




Relativistic generalization including the vector potential A

$$\Delta\Phi = \frac{q}{\hbar}\oint\left(V_0\,dt - \frac{A}{c}\cdot dx\right)$$

# ATOM INTERFEROMETER IN A FARADAY CAGE

Now consider an atom interferometer enclosed in a Faraday cage…




$$\Delta\tau = \frac{FD}{v_a - \Omega R} - \frac{CD}{v_a + \Omega R}$$

To the first order in $\frac{\Omega R}{v_a}$

$$\Delta\tau \cong \frac{2\pi R^2 \Omega}{v_a^2}$$

Then the phase shift due to potential is:

$$\Delta\Phi = \frac{q V_0 \Delta\tau}{\hbar}$$

In generalized form:

$$\Delta\Phi = \frac{2\, q V_0\, \Omega \,.\, A}{\hbar\, v_a^2}$$

# PHASE SHIFT SUMMARY:

$$\Delta\Phi_{sag} = \frac{4\pi}{h} m\Omega \, . \, A$$

Sagnac phase shift on an Atom interferometer…

$$\Delta\Phi_{pot} = \frac{2\, q V_0 \, \Omega \, . \, A}{\hbar \, v_a^2}$$

Atom interferometer in a Faraday cage

$$\frac{\Delta\Phi_{pot}}{\Delta\Phi_{sag}} = \frac{q V_o}{m v_a^2}$$

Ratio of the two phase shifts…

# IMPROVEMENT IN ROTATION SENSITIVITY

Our phase shift is now proportional to the voltage Vo of the cavity and inversely proportional to the square of the velocity of the atoms-unlike ordinary Sagnac interferometers-.

An example:

In a typical experiment were Cesium Atoms (AN=55, 132.9 amu) were used:

$$m = 2.2\times10^{-25}\ kg$$
$$v_a = 270\ m/\sec$$

$$\frac{\Delta\Phi_{pot}}{\Delta\Phi_{sag}} \approx 10\,V_o$$

Rotation sensitivity achieved:

$2\times10^{-8}\ rad/\sec$ in 1 Hz bandwidth…

**Orders of magnitude improvement is possible…**

# ACCELERATION SENSITIVITY

In general atom interferometers use diffraction gratings…hence they do not have a constant area as we assumed (change of diffraction angle with velocity) → Rotational response will have $1/v_a$ dependence…

Also, they have $1/v_a^2$ transverse acceleration sensitivity for the same reason…

Therefore reducing the velocity of atoms in ordinary atom interferometers is not desirable!

In our case however, we would have $1/v_a^3$ rotation sensitivity but the acceleration sensitivity would remain as $1/v_a^2$, therefore usage of slower atoms are beneficial.

# CONCLUSIONS

In general Sagnac Interferometer response does not depend on the wave velocity…

However, a matter wave interferometer in a Faraday cage will have a rotation response proportional inversely to the square of the velocity of the particles… which means by using slower particles we will have higher sensitivity…

Also the potential of the Faraday cage is an additional gain factor to increase the sensitivity…

Therefore we conclude that with our method it is possible to increase the rotation sensitivity of atom interferometers by orders of magnitude…